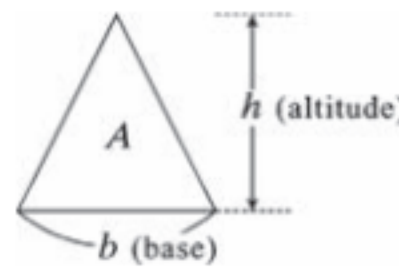
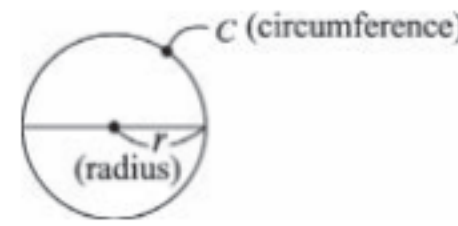
Topic	Formulas & Notes	Reference
Addition and Subtraction of Positive and Negative Numbers 1	$+5-2=+3, \quad +2-5=-3$ $-5+2=-3, \quad -2+5=+3$ '+' or '–' in front of a number is the sign of the number. '+' is read as 'plus', '–' is read as 'minus', and '=' is read as 'equals'. $0-5=-5, \quad -2-5=-7$	G21 G22a G24a
Addition and Subtraction of Positive and Negative Numbers 2	Ex. $\frac{1}{5}-2\frac{4}{5}=-2\frac{3}{5}$ $\frac{4}{5}-2\frac{1}{5}=\frac{4}{5}-1\frac{6}{5}=-1\frac{2}{5}$	G31a
Addition and Subtraction of Positive and Negative Numbers 3	When removing parentheses, remember to change the sign of the number within the parentheses if there is a negative sign in front of the parentheses. $+(-2)-(+4)=-2-4=-6$ $-(-2)+(+4)=2+4=6$	G46b
Multiplication / Division of Positive and Negative Numbers	For multiplication and division excluding 0, if there is: • an odd number of '–' signs (1, 3, 5, …), the answer is '–' • an even number of '–' signs (2, 4, 6, …), the answer is '+'	G63b G72b
Values of Algebraic Expressions	Replacing letters with numbers is called *substitution*. The result of the calculation is called the *value* of the expression. Ex. When $a=3$ and $b=2$, $6a-4b=6\times3-4\times2=18-8=10$	G101a G118a
Algebraic Expressions	(1) $2\times a=2a, \quad x\times y\times z=xyz$ (2) $1\div a=\frac{1}{a}, \quad b\div a=\frac{b}{a}$ (3) $a\times a=a^2, \quad a\times a\times b=a^2b$	
Simplifying Algebraic Expressions 1	Terms with the same variable can be combined as follows: Ex. $x+x+x=3x, \quad 3a+2a=5a, \quad 3a-5a=-2a$	G121a
Algebraic Notation	(1) $1x$ and $1a$ are written as x and a $-1x$ and $-1a$ are written as $-x$ and $-a$ (2) When simplifying algebraic expressions containing fractions, use improper fractions rather than mixed numbers. $\frac{7}{3}a$ is not changed to $2\frac{1}{3}a$	G121 G128a

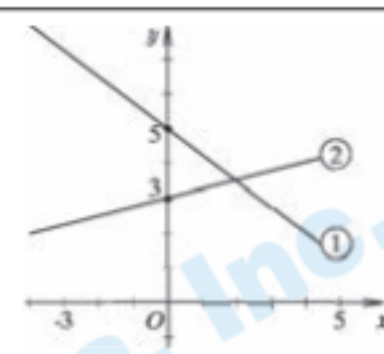
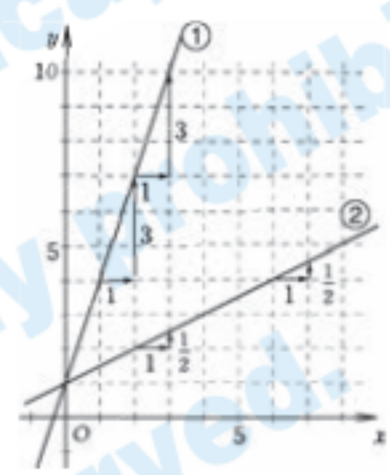
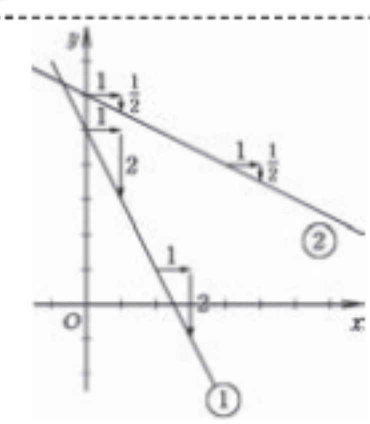
LEVEL G

Topic	Formulas & Notes	Reference
Simplifying Algebraic Expressions 2	Write answers in alphabetical order, (a, b, c or x, y, z) and in order of decreasing degree of the specific variable (x^2, x).	G135a
Simplifying Algebraic Expressions 3	Expressions such as $a(b + c)$ and $(b + c)a$ can be simplified as follows: $a(b + c) = ab + ac$ $(b + c)a = ab + ac$	G143a
Linear Equations 1	$2x = 6$ [Sol] $x = 6 \times \frac{1}{2}$ $x = 3$ To change the equation into the form "$x =$ a number", we transform $2x$ into x. We do this by multiplying both sides of the equation by $\frac{1}{2}$.	G161b
	$2x + 8 = 14$ $2x = 14 - 8$ The sign has to be changed when a term is transferred to the other side of the '=' sign. This is called *transposition*.	G163a
Linear Equations 3	To remove denominators, multiply each side of the equation by the *LCM*. Ex. $\frac{1}{2}x + \frac{1}{4} = \frac{1}{6}x + 1$ [Sol] Multiply each side by 12. The LCM of 2, 4 and 6 is 12. $\left(\frac{1}{2}x + \frac{1}{4}\right) \times 12 = \left(\frac{1}{6}x + 1\right) \times 12$ $6x + 3 = 2x + 12$ $6x - 2x = 12 - 3$ $4x = 9$ $x = \frac{9}{4}$	G181a
Linear Equations	In order for a given equation to remain a true statement, if an operation is performed on one side of the equation, the exact operation must also be performed on the other side of the equation. Given the equation $A = B$, (1) $A + m = B + m$ (2) $A - m = B - m$ (3) $Am = Bm$ (4) $\frac{A}{m} = \frac{B}{m}$ $(m \neq 0)$	

LEVEL H

Topic	Formulas & Notes	Reference
Transforming Equations 2	Area of a triangle: $A = \frac{1}{2}bh$ 	H36a
	Circumference of a circle: $C = 2\pi r$ (where $\pi \approx 3.14$) 	H36b
Simultaneous Linear Equations in Two Variables 1 *Addition or Subtraction Method*	$\begin{cases} 5x + 2y = 11 & \text{...①} \\ 3x + 2y = 5 & \text{...②} \end{cases}$ [Sol] ① - ②: $2x = 6$ $x = 3$ Substituting this into ①, $5 \times 3 + 2y = 11$ $2y = -4$ $y = -2$ Ans. $(x, y) = (3, -2)$ Procedure: 1. Number each equation. 2. Subtract one equation from the other to remove one variable. 3. Solve for x. 4. Substitute the value of x in one of the given equations and solve for y. 5. Write the answer.	H41a
Simultaneous Linear Equations in Two Variables 5 *Substitution Method*	$\begin{cases} y = 3x - 1 & \text{...①} \\ 5x - y = 5 & \text{...②} \end{cases}$ [Sol] Substituting ① into ②, $5x - (3x - 1) = 5$ $2x = 4$ $x = 2$ Substituting this into ①, $y = 3 \times 2 - 1 = 5$ Ans. $(x, y) = (2, 5)$	H81a
Inequalities 1	Given $A < B$, 1) $A + C < B + C$ or $A - C < B - C$. You can add the same number to or subtract the same number from both sides. 2) When $C > 0$, $AC < BC$ or $\frac{A}{C} < \frac{B}{C}$ You can multiply or divide both sides by the same number. 3) When $C < 0$, $AC > BC$ or $\frac{A}{C} > \frac{B}{C}$ When multiplying or dividing both sides by a negative number, you need to reverse the inequality symbol.	H121 H122 H123

LEVEL H

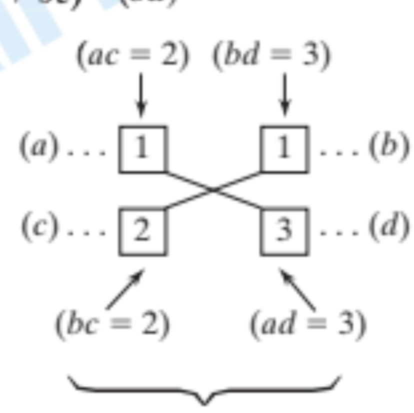
Topic	Formulas & Notes	Reference
Functions and Graphs 1	If the relation between x and y can be expressed as $y = mx$, the relation is called a *direct variation* and m is the *constant of variation*. The graph of a direct variation, $y = mx$, is a line that passes through the origin.	H147a
	If the relation between x and y can be expressed as $y = \frac{k}{x}$, the relation is called an *inverse variation* and k is the *constant of variation*.	H149a
Functions and Graphs 2	The *y-intercept* of a line is the y-coordinate of the point where the line intersects the y-axis. In line ①, the *y-intercept* is 5. In line ②, the *y-intercept* is 3. 	H151a
	The slope of a line is expressed by the ratio $\frac{rise}{run}$ (the steepness of the line). In line ①, the slope is 3; the line rises 3 units for every unit that it runs horizontally. In line ②, the slope is $\frac{1}{2}$; the line rises $\frac{1}{2}$ unit for every unit that it runs horizontally. 	H152a
	Lines ① and ② have negative slopes. The slope of line ① is -2 because the line falls 2 units for every unit that it runs horizontally. The slope of line ② is $-\frac{1}{2}$ because the line falls $\frac{1}{2}$ unit for every unit that it runs horizontally. 	H153a
	■ If the relation between x and y is expressed as $y = mx + b$, the relation is called a linear function. ■ In the above equation, m is the *slope*, and b is the *y-intercept*. ■ $y = mx + b$ is the equation of a line that passes through point $(0, b)$.	H156a

LEVEL H

Topic	Formulas & Notes	Reference
Functions and Graphs 3	The slope of a line that passes through the two points (a, b) and (c, d) can be found as follows: $$\text{Slope} = \frac{d-b}{c-a}$$	H165b
Functions and Graphs 4	An equation $ax + by + c = 0$ can be transformed to: $$y = -\frac{a}{b}x - \frac{c}{b}$$ The slope of the line of the above equation is $-\frac{a}{b}$ and the y-intercept is $-\frac{c}{b}$.	H171b
	Given an equation $ax + by + c = 0$, 1. If $a = 0$, the above equation is transformed to $y = -\frac{c}{b}$, its line passes through $\left(0, -\frac{c}{b}\right)$ and is parallel to the x-axis. The slope is 0. 2. If $b = 0$, the above equation is transformed to $x = -\frac{c}{a}$, its line passes through $\left(-\frac{c}{a}, 0\right)$ and is parallel to the y-axis. The slope is *undefined*.	
	The solution of simultaneous equations represents the coordinates of the point of intersection of the lines of the simultaneous equations.	H172b
	If the slopes of lines are equal, the lines are *parallel*. Line $y = mx + b$ is parallel to line $y = mx$.	H178a
Simplifying Monomials and Polynomials 1	Ex. $3a^2b \times 4a^3 = 12a^5b$ Procedure for simplifying: 1) Determine the sign of the answer. 2) Calculate the numbers in front of the letters. 3) Calculate the exponents.	H183a

[NOTES]

LEVEL I

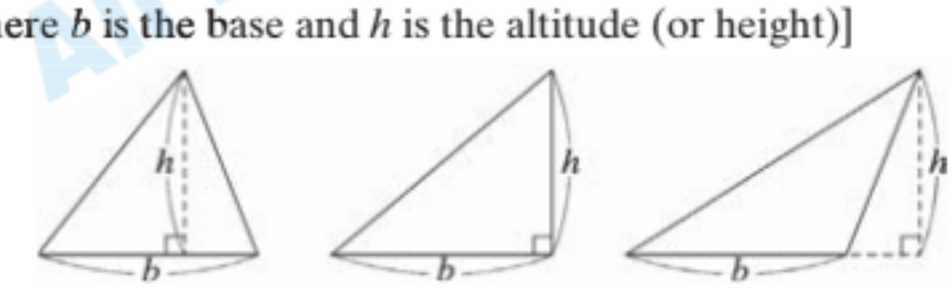
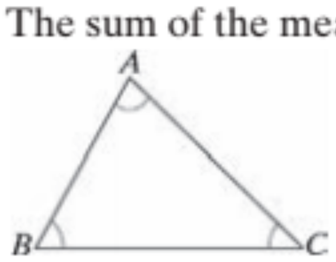
Topic	Formulas & Notes	Reference
Multiplication of Polynomials	$(a+b)(x+y) = \underset{①}{\underline{ax}} + \underset{②}{\underline{ay}} + \underset{③}{\underline{bx}} + \underset{④}{\underline{by}}$	I11a
Multiplication using Formulas	$(a+b)^2 = a^2 + 2ab + b^2$	I21a
	$(a-b)^2 = a^2 - 2ab + b^2$	I22a
	$(a+b)(a-b) = a^2 - b^2$	I26a
	$(x+a)(x+b) = x^2 + (a+b)x + ab$	I27a
	$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$	I29a
Factorization 1	$ax + ay = a(x+y)$	I31a
	$(a+b)x + (a+b)y = (a+b)(x+y)$	I33a
	$a^2 + 2ab + b^2 = (a+b)^2$ $a^2 - 2ab + b^2 = (a-b)^2$	I36a
Factorization 2 *Difference of Two Squares*	$a^2 - b^2 = (a+b)(a-b)$	I41a
	$x^2 + (a+b)x + ab = (x+a)(x+b)$	I44a
Factorization 3 *Cross Multiplication Method*	$acx^2 + (ad+bc)x + bd = (ax+b)(cx+d)$ Ex. $\underset{(ac)}{2}x^2 + \underset{(ad+bc)}{5}x + \underset{(bd)}{3} = (x+1)(2x+3)$  $(ad + bc = 3 + 2 = 5)$ This method can be used to find the coefficients a, b, c, and d. We estimate the coefficients so that $ac = 2$, $bd = 3$, and $ad + bc = 5$.	I51a
Factorization 5	$y - x = -x + y = -(x-y)$	I71b
	$x^2(a-b) + y^2(b-a)$ $= x^2(a-b) - y^2(a-b)$ $= (a-b)(x^2-y^2)$ $= (a-b)(x+y)(x-y)$ To find the common factor, try changing the signs and order of the terms.	I72a
	$ax + ay + bx + by = a(x+y) + b(x+y)$ $= (x+y)(a+b)$	I76a

LEVEL I

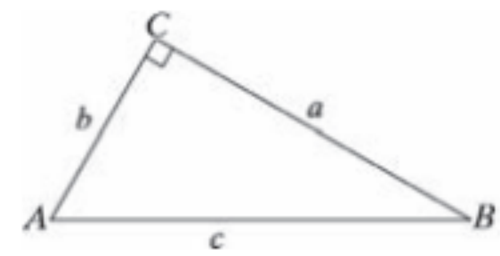
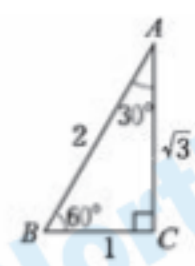
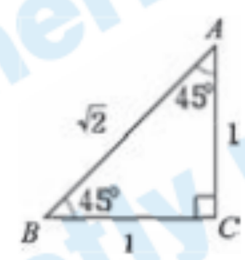
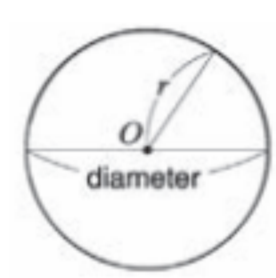
Topic	Formulas & Notes	Reference
Square Roots 1	A *square root* of a number is a number that when squared is equal to the given number. Ex. The square roots of 25 are 5 and -5	I81a
	$\sqrt{a}$ represents the *positive square root* of a number a. (The negative square root is represented by $-\sqrt{a}$.) Ex. $\sqrt{144} = 12$	I82a
Square Roots 2 *Multiplication of Square Roots*	For any positive numbers a and b, $\sqrt{a}\sqrt{b} = \sqrt{ab}$.	I91a
Prime Factorization	1) A *prime number* is an integer greater than 1 that has no positive integral factors other than itself and 1. Ex.(2, 3, 5, 7, 11, 13, 17...) 2) The factorization of an integer over its positive prime numbers is called *prime factorization*. Ex. $45 = 9 \times 5 = 3 \times 3 \times 5 = 3^2 \times 5$	I94a
Addition and Subtraction of Square Roots	$\sqrt{5} + \sqrt{5} = 2\sqrt{5}$ $\sqrt{18} + \sqrt{50} = 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$ $\sqrt{12} - \sqrt{48} = 2\sqrt{3} - 4\sqrt{3} = -2\sqrt{3}$	I99a
Square Roots 3 *Division of Square Roots*	For any positive numbers a and b, $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$	I106a
Rationalizing the Denominator	$\frac{b}{\sqrt{a}} = \frac{b\sqrt{a}}{\sqrt{a}\sqrt{a}} = \frac{b\sqrt{a}}{a}$ $(a > 0)$ Multiply both the denominator and the numerator by the radical in the denominator, $\sqrt{a}$.	I107a
Quadratic Equations 1	Given a quadratic equation of the form $ax^2 + bx + c = 0 \ \ (a \neq 0)$, If the left side of the equation can be factored as $(dx + e)(fx + g) = 0$, then the solution of the equation is: $x = -\frac{e}{d}, \ \ x = -\frac{g}{f}$	I111a

[NOTES]

LEVEL I

Topic	Formulas & Notes	Reference
Quadratic Equations 2 *Completing the Square*	$2x^2 - 3x - 1 = 0$ [Sol] $x^2 - \frac{3}{2}x = \frac{1}{2}$ Transpose the numerical (constant) term to the right side of the equation, and divide both sides of the equation by the coefficient of the square term, in this case 2. $\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} = \frac{1}{2}$ Add a constant term to the left side so that it includes the square of a polynomial. To keep the equation a true statement, subtract the term from the same side. $\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$ (Take the coefficient of the x-term, divide it by 2, and square the result.) $\left(-\frac{3}{2}\right) \cdot \frac{1}{2} = -\frac{3}{4}, \left(-\frac{3}{4}\right)^2 = \frac{9}{16}$ $\vdots$ (continue solving as usual)	I124a I129a
Quadratic Equations 3 *Quadratic Formula*	Given $ax^2 + bx + c = 0 \quad (a \neq 0)$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	I131a
Graphs of Quadratic Functions	1) The graph of $y = ax^2$ $(a \neq 0)$: Vertex: $(0, 0)$, axis of symmetry: line $x = 0$ (the y-axis) 2) The graph of $y = a(x - p)^2 + q$ $(a \neq 0)$: Vertex: (p, q), axis of symmetry: line $x = p$ 3) The graph of $y = ax^2 + bx + c$ $(a \neq 0)$ $= a\left(x + \dfrac{b}{2a}\right)^2 - \dfrac{b^2 - 4ac}{4a}$ Vertex: $\left(-\dfrac{b}{2a}, -\dfrac{b^2 - 4ac}{4a}\right)$, axis of symmetry: line $x = -\dfrac{b}{2a}$	I150 I153b
The Pythagorean Theorem 1 *Areas of Triangles*	$A = \frac{1}{2}bh$ [where b is the base and h is the altitude (or height)] 	I171a
The Triangle-Sum Theorem	The sum of the measures of the angles of a triangle is 180 °.  $m\angle A + m\angle B + m\angle C = 180°$	I171b

LEVEL I

Topic	Formulas & Notes	Reference
Pythagorean Theorem	In a *right triangle*, the square of the length of the *hypotenuse* (c) equals the sum of the squares of the other two sides (a and b) . If $m\angle C = 90°$, $c^2 = a^2 + b^2$ 	I172a
Ratios and Proportions	The expression $a : b$ represents the *ratio* of a to b. $a : b = \frac{a}{b} \quad (b \neq 0)$ If $a : b = c : d$, then $ad = bc$ (Note: $a : b = c : d$ is called a *proportion* and can be written as $\frac{a}{b} = \frac{c}{d} \quad (b \neq 0, d \neq 0)$)	I179a I179b
Special Right Triangles	30°-60°-90°  $1 : \sqrt{3} : 2$ Ratio of the length of each side to the other 45°-45°-90°  *(isosceles right triangle)* $1 : 1 : \sqrt{2}$ Ratio of the length of each side to the other	I180a
The Pythagorean Theorem 2 *Circumference and Area of Circles*	For a circle with radius r, ($\pi \approx 3.14$) Circumference, $C = 2\pi r$ Area, $A = \pi r^2$ 	I189a
Area of a Sector	For a circle with radius r, the area, A, of a sector whose central angle measures $m°$ is given by: $A = \frac{m}{360}\pi r^2$ 	I189b
The Pythagorean Theorem 3 *Distance*	The distance (AB) between points $A(a, b)$ and $B(c, d)$: $AB = \sqrt{(c - a)^2 + (d - b)^2}$	I191a

LEVEL J

Topic	Formulas & Notes	Reference
Expansion of Polynomial Products *Laws of Exponents*	For any number a and b, when m and n are positive integers: $a^m \times a^n = a^{m+n}$, $(a^m)^n = a^{m \times n}$, $(ab)^m = a^m b^m$	J1a
Distributive Property	For any numbers a, b, and c: $c(a+b) = ca + cb$	
	$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$	J4b
	$(a+b)(a-b) = a^2 - b^2$	J5a
	$(x+a)(x+b) = x^2 + (a+b)x + ab$	J6a
	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	J7a J8a
	$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	J7b J8a
	$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$	J9b
Factorization 1	$x^2 + (a+b)x + ab = (x+a)(x+b)$	J11a
	$a^2 + 2ab + b^2 = (a+b)^2$ $a^2 - 2ab + b^2 = (a-b)^2$	J15a
Difference of Two Squares	$a^2 - b^2 = (a+b)(a-b)$	J17a
Treating Multiple Terms as a Single Quantity	Given — Terms to be treated as single quantities $(x+y)^2 - 3(x+y) - 10$ — $(x+y)$ $(a+b)^2 - (c+d)^2$ — $(a+b)$ and $(c+d)$ $x^4 - y^4 = (x^2)^2 - (y^2)^2$ — x^2 and y^2	 J11b J18a J19a
Factorization 2 *Factoring out '−1'*	$(b-a) = -(a-b)$ $(b-a)^2 = (a-b)^2$ $(b-a)^3 = -(a-b)^3$	J22b J23a J25a
Factorization by Grouping	$ax + ay + bx + by = a(x+y) + b(x+y) = (x+y)(a+b)$, or $ax + ay + bx + by = x(a+b) + y(a+b) = (a+b)(x+y)$	J26a
	$\overset{\wedge}{x^3} + \overset{\wedge}{4x^2} + \overset{*}{2x} + \overset{*}{8} = x^2(x+4) + 2(x+4) = (x+4)(x^2+2)$ (Note: here the symbols ^ and * are used to mark the terms that are grouped together in the intermediate step of this example.)	J28a
Factorization 3 *Arranging in Standard Polynomial Form*	Given $x^2 + 5xy + 6x + 6y^2 + 13y + 5$, With x as the variable: $x^2 + (5y+6)x + (6y^2 + 13y + 5)$ With y as the variable: $6y^2 + (5x+13)y + (x^2 + 6x + 5)$	J34a

LEVEL J

Topic	Formulas & Notes	Reference
Factorization 4 *Sum and Difference of Two Cubes*	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	J47a J48a
Factorization 5 *Factoring Fourth Degree Polynomials*	$x^4 - 6x^2 + 1$ $= x^4 - 2x^2 + 1 - 4x^2$ $= (x^2 - 1)^2 - (2x)^2$ $\vdots$ (continue factoring as usual) Break up the x^2 term into two terms forming a binomial square minus a monomial square; i.e., *the difference of two squares.*	J51a
Treating Multiple Terms as a Single Quantity	$(x^2 + x - 6)(x^2 + x - 2) + 3 \longleftarrow$ Let $x^2 + x = A$ $= (A - 6)(A - 2) + 3$ $= A^2 - 8A + 15 = (A - 3)(A - 5) = (x^2 + x - 3)(x^2 + x - 5)$	J53a
Fractional Expressions	$\dfrac{mA}{mB} = \dfrac{A}{B}$	J61a
	$\dfrac{A}{B} \times \dfrac{C}{D} = \dfrac{A \times C}{B \times D}, \quad \dfrac{A}{B} \div \dfrac{C}{D} = \dfrac{A}{B} \times \dfrac{D}{C} = \dfrac{A \times D}{B \times C}$	J64a
	$\dfrac{A}{C} \pm \dfrac{B}{C} = \dfrac{A \pm B}{C}$	J66a
	$\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{b}{ab} + \dfrac{a}{ab} = \dfrac{a + b}{ab}$	J66b
Irrational Numbers 1 *Rationalizing the Denominator*	Ex. $\dfrac{\sqrt{3}}{\sqrt{5}} = \dfrac{\sqrt{3} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \dfrac{\sqrt{15}}{5}$	J74a
	$\dfrac{c}{\sqrt{a} \pm \sqrt{b}} = \dfrac{c(\sqrt{a} \mp \sqrt{b})}{(\sqrt{a} \pm \sqrt{b})(\sqrt{a} \mp \sqrt{b})} = \dfrac{c(\sqrt{a} \mp \sqrt{b})}{a - b} \quad (a > 0, b > 0)$	J75a
Double Radicals	$\sqrt{7 + 2\sqrt{10}} = \sqrt{5} + \sqrt{2}$ $\uparrow$ 5 + 2 $\quad\uparrow$ 5 × 2 (The two resulting radicands must add to give 7, and multiply to give 10.)	J77b
Irrational Numbers 2	$\sqrt{a^2} = \begin{cases} a \text{ (when } a \geq 0) \\ -a \text{ (when } a < 0) \end{cases}$ Note: $[\sqrt{0} = 0]$	J86b
	$\sqrt{(a - b)^2} = \begin{cases} a - b & \text{(when } a \geq b) \\ -(a - b) & \text{(when } a < b) \end{cases}$	J87b
Quadratic Equations 1 *Quadratic Formula I*	When $ax^2 + bx + c = 0 \ (a \neq 0), \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	J97b

LEVEL J

Topic	Formulas & Notes	Reference
Quadratic Equations 2 *Quadratic Formula II*	When $ax^2 + 2b'x + c = 0 \ \ (a \neq 0)$, $\ x = \dfrac{-b' \pm \sqrt{b'^2 - ac}}{a}$ Use *Quadratic Formula II* to solve quadratic equations when the coefficient of x is an even number.	J102a
Quadratic Equations and Complex Numbers	Positive and negative numbers, and zero, belong to the set of *real numbers*. All positive and negative numbers become positive when squared. 0 becomes 0 when squared. The *imaginary number*, i , is not a real number, and becomes negative when squared.	J111a
Imaginary Number	$i^2 = -1 \quad (\sqrt{-1} = i)$	
	When $a \geq 0$, $\ \sqrt{-a} = \sqrt{a}i$	J111b
	$\sqrt{a}\sqrt{b} = \sqrt{ab}$ (unless $a < 0$ and $b < 0$)	J112b
	$\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$ (unless $a > 0$ and $b < 0$)	J113a
Complex Numbers	$3 + 2i$ and $4 - 3i$ are *complex numbers*. $a + bi$ consists of real and imaginary parts; a and b are real parts. $a + bi$ is a real number when $b = 0$ (since $a + 0 \cdot i = a$) When $b \neq 0$, $a + bi$ is a complex number.	J113b
	$a + bi = c + di \Leftrightarrow a = c, b = d$ (Where a, b, c, d $a + bi = 0 \Leftrightarrow a = 0, b = 0$ are real numbers.)	J115b
	If a quadratic equation in the form $ax^2 + bx + c = 0$ has complex roots, and one of them is $x = \alpha + \beta i$, then the other one is $x = \alpha - \beta i$.	J119b
Discriminant	Given $ax^2 + bx + c = 0$, $\ D = b^2 - 4ac$ When $D > 0$, there are two different real number solutions. When $D = 0$, there is one repeated solution. When $D < 0$, there are two different complex number solutions.	J121b
	Given $ax^2 + 2b'x + c = 0$, $\ D' = \dfrac{D}{4} = b'^2 - ac$	J123a
Root-Coefficient Relationship	Given $ax^2 + bx + c = 0$ $(a \neq 0)$, assuming the roots are α and β: $\alpha + \beta = -\dfrac{b}{a}, \quad \alpha\beta = \dfrac{c}{a}$	J131b
Dividing Polynomials	In the division of $(x^3 - x^2 - 1) \div (x - 2)$ the quotient is $x^2 + x + 2$ with remainder 3. This relationship may be written as: $x^3 - x^2 - 1 = (x - 2)(x^2 + x + 2) + 3$	J154a

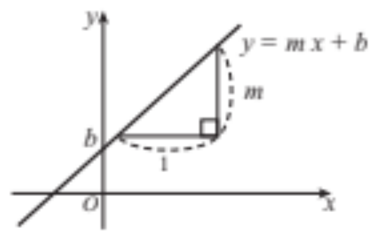
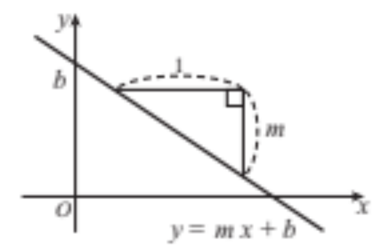
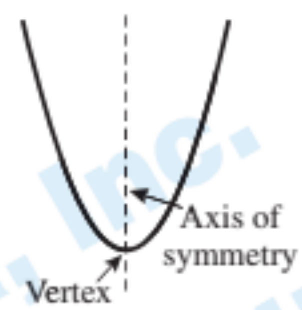
LEVEL J

Topic	Formulas & Notes	Reference
Remainder Theorem	If a polynomial $P(x)$ is divided by a term $(x-a)$, we can express the relation with the quotient $Q(x)$, and the remainder R as: $$P(x) = (x-a)Q(x) + R$$ When $x = a$, $P(a) = (a-a)Q(x) + R = R$. Therefore,	J162b
Remainder Theorem	When polynomial $P(x)$ is divided by $(x-a)$, the remainder is $P(a)$.	J163a
	The *Remainder Theorem* can only be used when dividing by a linear expression. When dividing by a quadratic expression, use long division.	J164b
General Form of a Remainder	-When the divisor is a second-degree expression (a quadratic) the remainder must be of degree one or less, in the form: $ax + b$ -When the divisor is a third-degree expression (a cubic) the remainder must be of degree two or less, in the form: $ax^2 + bx + c$	J165b
Factor Theorem *Factor Theorem*	When $P(a) = 0$, it means that the polynomial $P(x)$ has a factor, $(x-a)$. Therefore, a polynomial, $P(x)$, has a factor $(x-a)$, if and only if, $\Leftrightarrow$ $P(a) = 0$.	J172a
	Given a polynomial $P(x)$, to determine a value, a, at which $P(a) = 0$, we first check the coefficient of the highest power of the variable in the polynomial. ▶ If it is 1, we try a number that is a factor of the constant in the polynomial. ▶ If it is greater than 1, we try a number such that: - Its numerator is a factor of the constant in the polynomial. And, - Its denominator is a factor of the coefficient of the highest power of the variable in the polynomial.	J172a J174a
Proof of Identities and Equalities *Coefficient Comparison Method*	There are 2 ways to determine the value of the coefficients to show that an equality is an identity: ① Given $(ax+b)(x+1) = 3x^2 + 5x + 2$, $ax^2 + (a+b)x + b = 3x^2 + 5x + 2$ Expand the LHS and arrange with x as the variable. Matching the coefficients of x^2, x, and the constants of the LHS and RHS: $a = 3$, $a + b = 5$, $b = 2$ (Ans. $a = 3$, $b = 2$)	J181a
Value Substitution Method	② Given $x^2 = a(x-1)(x-2) + b(x-1) + c$, Substitute values of x to both sides of the equation. When $x = 1$, $1 = c$ …① When $x = 2$, $4 = b + c$ …② When $x = 3$, $9 = 2a + 2b + c$ …③ From ①, ②, and ③, (Ans. $a = 1$, $b = 3$, $c = 1$)	J182b

Topic	Formulas & Notes	Reference
	In order for $ax^2 + bx + c = 0$ to be an identity in x, the following must be true: $a = 0, b = 0, c = 0$.	J182b
	Given an equation, If we show that LHS − RHS = 0 Then we know that LHS = RHS	J188b
Proof of Equalities and Inequalities	Given an equation, If we show that LHS − RHS > 0 Then we know that LHS > RHS	J194a
	Given an equation, If we show that LHS − RHS ≥ 0 Then we know that LHS ≥ RHS	J194b
Arithmetic Mean Geometric Mean	Given $a > 0, b > 0$: $\frac{a+b}{2} \geq \sqrt{ab}$ [i.e., (arithmetic mean) ≥ (geometric mean)] The LHS equals the RHS when $a = b$.	J197b

[NOTES]

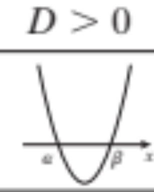
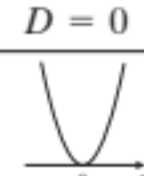
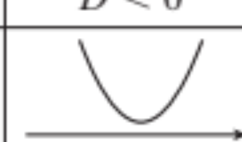
LEVEL K

Topic	Formulas & Notes	Reference
Review of Linear Functions *Equation of a Linear Function*	$y = mx + b$ where m is the *slope* and b is the *y-intercept*. When $m > 0$  When $m < 0$ 	K3
Slope of a Line	Given two points with coordinates (x_1, y_1) and (x_2, y_2), $m = \dfrac{y_2 - y_1}{x_2 - x_1} \quad \left(= \dfrac{\textit{the change in } y}{\textit{the change in } x}\right)$	K5a
Review of Quadratic Functions	The graph of a quadratic function is called a *parabola*. The parabola has a symmetrical axis called the *axis* of the parabola, or the *axis of symmetry*. The point of intersection of the axis of symmetry and the parabola is called the *vertex* of the parabola. 	K11b
Equation of a Quadratic Function (a, b, c Format) *&* *(p, q Format)*	• A function that can be written in the form: $y = ax^2 + bx + c$ (where $a \neq 0$) is called a *quadratic function*. • If a, p, and q are constants and $a \neq 0$, given a function of the form $y = a(x - p)^2 + q$ the *axis of symmetry* is $x = p$, and the coordinates of the *vertex* are (p, q).	K11a K13b
Changing Format *a, b, c to p, q*	To change the format of a quadratic equation from a, b, c to p, q, apply the method of *completing the square*. Ex. $y = x^2 - 2x + 3$ [Sol] $= (x^2 - 2x) + 3$ $= (x^2 - 2x + 1) - 1 + 3$ $= (x - 1)^2 + 2$	K14a
Quadratic Functions and Graphs *Translations*	The graph of $y = a(x - p)^2 + q$ is a *translation* of the graph of $y = ax^2$, p units along the x-axis, and q units along the y-axis.	K27b
Determining Equations of Quadratic Functions	When the graph of a parabola crosses the x-axis at points $(\alpha, 0)$ and $(\beta, 0)$ the equation of the function can be written as: $y = a(x - \alpha)(x - \beta)$	K35b

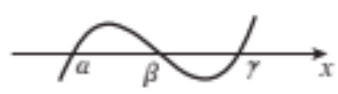
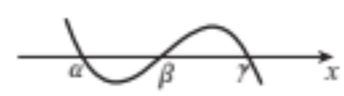
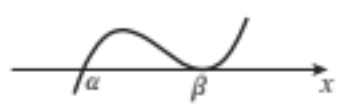
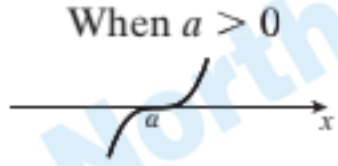
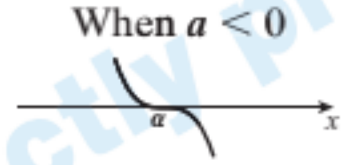
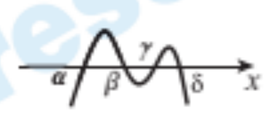
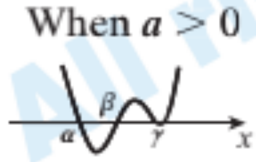
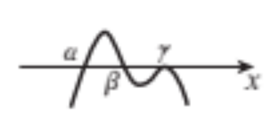
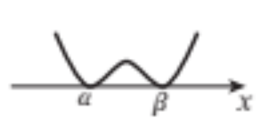
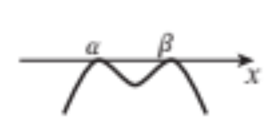
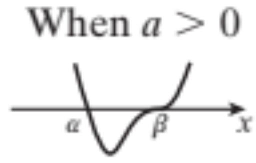
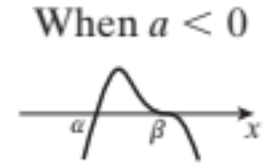
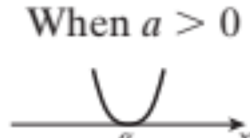
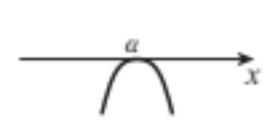
[NOTES]

LEVEL K

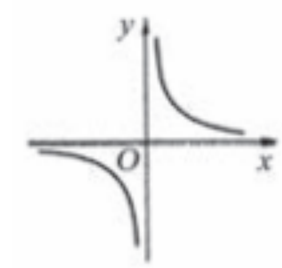
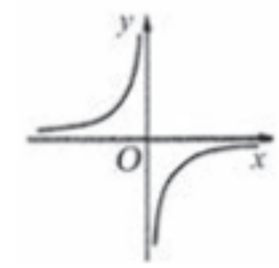
Topic	Formulas & Notes	Reference
Maxima and Minima of Quadratic Functions 1	Given the quadratic function $y = ax^2 + bx + c$: • When $a > 0$, the parabola opens upward, and it has a minimum value at the vertex. (It does not have a maximum value.) The minimum value is $-\dfrac{b^2 - 4ac}{4a}$, at $x = -\dfrac{b}{2a}$. • When $a < 0$, the parabola opens downward, and it has a maximum value at the vertex. (It does not have a minimum value.) The maximum value is $-\dfrac{b^2 - 4ac}{4a}$, at $x = -\dfrac{b}{2a}$.	K41b
Maxima and Minima of Quadratic Functions 2	Given a quadratic function $y = ax^2 + bx + c$ with domain $\alpha \leq x \leq \beta$, (comparing the location of the specific domain to the location of the axis of symmetry, $x = -\dfrac{b}{2a}$), the maximum and minimum values of the graph $y = f(x)$ are defined at the left end of the domain, at the vertex, or at the right end of the domain.	K52 K55
Quadratic Functions and Equations	Given a quadratic function of the form $y = ax^2 + bx + c$, when the graph has *common points* with the x-axis at: 2 points, $D = b^2 - 4ac > 0$ 1 point, $D = b^2 - 4ac = 0$ 0 points, $D = b^2 - 4ac < 0$	K72b
Quadratic Functions and Inequalities	• The part of the graph of $y = ax^2 + bx + c$ at which $ax^2 + bx + c > 0$, consists of all the values of x at which the graph is above the x-axis. • The part of the graph of $y = ax^2 + bx + c$ at which $ax^2 + bx + c < 0$, consists of all the values of x at which the graph is below the x-axis.	K81a
	The relationship between the graph of the quadratic function $y = ax^2 + bx + c$, and the quadratic inequalities: $ax^2 + bx + c > 0, \quad ax^2 + bx + c \geq 0$ $ax^2 + bx + c < 0, \quad ax^2 + bx + c \leq 0$ can be summarized as follows. (Assume $a > 0$.)	K86b

The sign of D	$D > 0$	$D = 0$	$D < 0$
The graph of $y = ax^2 + bx + c$			
solving $ax^2 + bx + c > 0$	$x < \alpha, \beta < x$	$x < \alpha, \alpha < x$	All real numbers
solving $ax^2 + bx + c \geq 0$	$x \leq \alpha, \beta \leq x$	All real numbers	All real numbers
solving $ax^2 + bx + c < 0$	$\alpha < x < \beta$	No solution	No solution
solving $ax^2 + bx + c \leq 0$	$\alpha \leq x \leq \beta$	$x = \alpha$	No solution

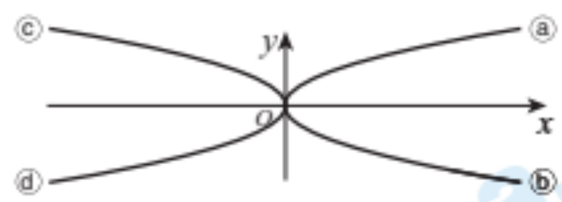
LEVEL K

Topic	Formulas & Notes	Reference
Quadratic Functions and Solutions of Quadratic Equations	Given a quadratic equation $ax^2 + bx + c = 0$ $(a > 0)$ that has 2 solutions α, β $(\alpha < \beta)$, you can find the signs of α, β by determining the signs of D, $-\dfrac{b}{2a}$, and $f(0)$. • When $D > 0$, $-\dfrac{b}{2a} > 0$, $f(0) > 0$, then $\alpha > 0, \beta > 0$. • When $D > 0$, $-\dfrac{b}{2a} < 0$, $f(0) > 0$, then $\alpha < 0, \beta < 0$. • When $D > 0$, $f(0) < 0$, then $\alpha < 0, \beta > 0$.	K91b
Higher Degree Functions *Rough Graphs of Cubic Functions*	• $y = a(x-\alpha)(x-\beta)(x-\gamma)$, (where $\alpha < \beta < \gamma$): When $a > 0$  When $a < 0$  • $y = a(x-\alpha)(x-\beta)^2$, (where $\alpha < \beta$): When $a > 0$  When $a < 0$  • $y = a(x-\alpha)^3$: When $a > 0$  When $a < 0$ 	K104b
Rough Graphs of Quartic Functions	• $y = a(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$ $(\alpha < \beta < \gamma < \delta)$: When $a > 0$  When $a < 0$  • $y = a(x-\alpha)(x-\beta)(x-\gamma)^2$ $(\alpha < \beta < \gamma)$: When $a > 0$  When $a < 0$  • $y = a(x-\alpha)^2(x-\beta)^2$ $(\alpha < \beta)$: When $a > 0$  When $a < 0$  • $y = a(x-\alpha)(x-\beta)^3$ $(\alpha < \beta)$: When $a > 0$  When $a < 0$  • $y = a(x-\alpha)^4$: When $a > 0$  When $a < 0$ 	K107b

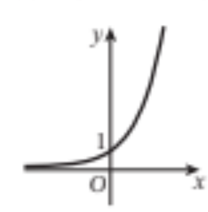
LEVEL K

Topic	Formulas & Notes	Reference
Graphs of Fractional Functions 1	On the graph of $y = \frac{a}{x}$ (where $a \neq 0$) the *asymptotes* are: the y-axis and the x-axis When $a > 0$  When $a < 0$ 	K122
Asymptotes and Translations	Given a fractional function of the general form $y = \frac{k}{x-p} + q \quad (k \neq 0)$, the equations of the graph's asymptotes are $x = p$, $y = q$. This graph has been translated from the graph of $y = \frac{k}{x}$, p units along the x-axis, and q units along the y-axis.	K124a K127b
Identifying Asymptotes	In order to identify the asymptotes easily, you can transform an equation as follows: $y = \frac{x+1}{x-2} = \frac{(x-2)+3}{x-2} = 1 + \frac{3}{x-2}$	K125a
x-intercept	To find the x-coordinate of the point of intersection of the graph $y = \frac{ax+b}{cx+d}$ and the x-axis, you can solve the equation $ax + b = 0$ (which is the numerator = 0). (For example, given $y = \frac{x+1}{x-2}$, solving the equation $x + 1 = 0$ we get $x = -1$, which is the x-*intercept*, i.e. the x-coordinate of the x-axis point of intersection.)	K125b
Graphs of Fractional Functions 2 *Asymptotes*	The equations of the asymptotes of the graph $y = ax + \frac{b}{x}$ are: $x = 0, \quad y = ax$	K137a
Fractional Equations and Inequalities	When solving fractional equations, we must always check each solution and exclude any one that makes the denominator of the original equation 0. $A = B \rightleftarrows AC = BC \quad (AC = BC \rightleftarrows A = B$ or $C = 0)$	K144
Graphs of Irrational Functions	• An irrational function of the form $y = \sqrt{k(x-p)} + q$ is a translation of $y = \sqrt{kx}$, p units along the x-axis, and q units along the y-axis. • An irrational function of the form $y = -\sqrt{k(x-p)} + q$ is a translation of $y = -\sqrt{kx}$, p units along the x-axis, and q units along the y-axis.	K157b

LEVEL K

Topic	Formulas & Notes	Reference
Domain & Range	• Given an irrational function of the general form $y = \sqrt{k(x-p)} + q$, the domain and range of the function are as follows: When $k > 0$, Domain: $x \geq p$, Range: $y \geq q$ When $k < 0$, Domain: $x \leq p$, Range: $y \geq q$	K160b
Translations	• An irrational function of the general form $y = \sqrt{k(x-p)} + q$ is a translation of the graph of $y = \sqrt{kx}$, p units along the x-axis, and q units along the y-axis.	
Rough Graphs	• The rough graphs of some common irrational functions are shown below. a $y = \sqrt{x}$, b $y = -\sqrt{x}$, c $y = \sqrt{-x}$, d $y = -\sqrt{-x}$ 	
Irrational Equations and Inequalities	When solving irrational equations we must always check the solutions by using either of the following two methods: • The solution is the x-coordinate of the common point on the graph. OR, • When substituting the solution into the original equation, LHS = RHS. $A = B \rightleftarrows A^2 = B^2 \quad (A^2 = B^2 \rightleftarrows A = B$ or $A = -B)$	K164b
Exponential Functions *Definition of a^0 and a^{-n}*	1. If $a^m \times a^n = a^{m+n}$ is true even when $m = 0$, then, $a^0 \times a^n = a^n$ so, $a^0 = 1$. 2. If $a^m \div a^n = a^{m-n}$ is true even when $m = 0$, then, $1 \div a^n = a^{-n}$ so, $a^{-n} = \dfrac{1}{a^n}$.	K171a
Laws of Exponents	When $a \neq 0$, $b \neq 0$, $a^0 = 1$ $a^{-n} = \dfrac{1}{a^n}$ $a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{mn}$ $(ab)^m = a^m b^m$	K171b
The n^{th} Root of a	The number which when raised to the power of n is equal to a is called the n^{th} *root of a*. ($\sqrt[3]{8}$ is called the 3rd root of 8, $\sqrt[3]{-8}$ is the 3rd root of -8, $\sqrt[n]{a}$ is the n^{th} root of a.)	K173b

LEVEL K

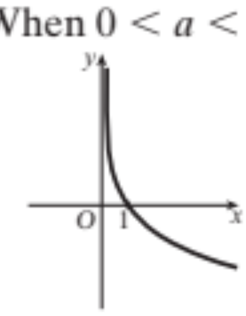
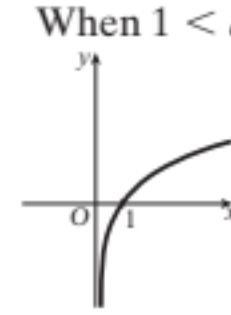
Topic	Formulas & Notes	Reference
The Number of *n^{th} Roots*	• When n is an even number (square roots, 4th roots . . .), When $a > 0$, there are 2 n^{th} roots (ex. $\sqrt[n]{a}$, $-\sqrt[n]{a}$). When $a < 0$, there are no real number n^{th} roots. • When n is an odd number (cube roots, 5th roots . . .), There is only 1 real number n^{th} root. (However, if we include imaginary numbers, there are 3 cube roots, 4 fourth roots, etc.)	K174a
Laws of Exponents	When $a > 0$, $b > 0$ and m, n, and p are positive integers: $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$ $\quad \sqrt[mp]{a^{np}} = \sqrt[m]{a^n}$ $\quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$ $\quad \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$	K174b
	$\sqrt[3]{-16} = -\sqrt[3]{16}$, $\sqrt[5]{-32} = -\sqrt[5]{32} = -2$ When n is an odd number, $\sqrt[n]{-a} = -\sqrt[n]{a}$ (where $a > 0$).	K176a
Laws of Exponents	When $a > 0$, m is an integer, and n is a positive integer, $a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \left(a^{\frac{1}{n}} = \sqrt[n]{a}\right)$	K177a
Graphs of Exponential Functions	When $a > 0$ and $a \neq 1$, a function of the form $y = a^x$ is called an *exponential function* of x, where a is the *base*. The graph of $y = a^x$ passes through point (0, 1), and the x-axis is the asymptote. When $0 < a < 1$  When $1 < a$ 	K182b
Translations	The graph of $y = a^{x-p} + q$ (where $a > 0$ and $a \neq 1$) is a translation of the graph of $y = a^x$, p units along the x-axis, and q units along the y-axis.	K185b
Comparing Numbers	• Given the exponential function $y = a^x$, when $a > 1$, as x increases, y increases. Therefore, $p < q \Leftrightarrow a^p < a^q$ $\quad$ ($p < q$ if and only if $a^p < a^q$) • Given the exponential function $y = a^x$, when $0 < a < 1$, as x increases, y decreases. Therefore, $p < q \Leftrightarrow a^p > a^q$ $\quad$ ($p < q$ if and only if $a^p > a^q$) • Given the exponential functions $y = a^x$ and $y = b^x$, when $a > 0$, $b > 0$, and $p > 0$, $a < b \Leftrightarrow a^p < b^p$ $\quad$ ($a < b$ if and only if $a^p < b^p$)	K187a K187b K188a

LEVEL K

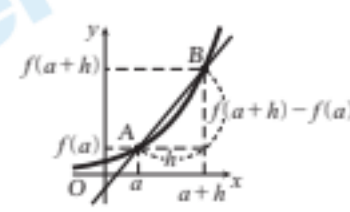
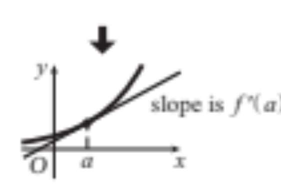
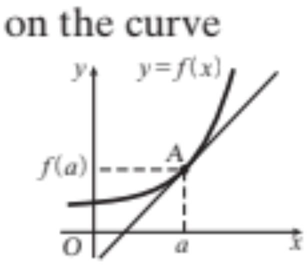
Topic	Formulas & Notes	Reference
Exponential Equations and Inequalities *Solving Methods*	• If we let $a^x = X$ (where $X > 0$) and can rewrite the equation in terms of X, then we solve for X. • When $a^x = a^m$, then $x = m$	K194a
	• When $a > 1$, then $a^x > a^m \rightleftarrows x > m$ • When $0 < a < 1$, then $a^x > a^m \rightleftarrows x < m$	K196

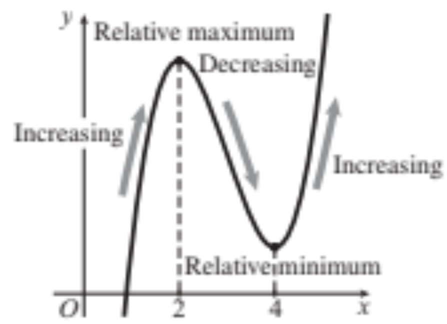
[NOTES]

LEVEL L

Topic	Formulas & Notes	Reference
Logarithmic Functions	$a^n = N$ can be written as $\log_a N = n$	L1a
	Given $\log_a N$ (where $a > 0, a \neq 1, N > 0$): a is called the *base*, and N is called the *antilogarithm*. $\log_a N$ is called the *logarithm of N, to the base a.*	L1b
Properties of Logarithms	When $a > 0, a \neq 1, M > 0, N > 0$, **1** $\log_a 1 = 0, \quad \log_a a = 1, \quad \log_a a^m = m$ **2** $\log_a MN = \log_a M + \log_a N$ **3** $\log_a \frac{M}{N} = \log_a M - \log_a N, \quad \left[\log_a \frac{1}{N} = -\log_a N\right]$ **4** $\log_a M^n = n\log_a M \quad \left[\log_a \sqrt[n]{M} = \frac{1}{n}\log_a M\right]$	L5a
Graphs of Logarithmic Functions *Logarithmic Base Conversion Formula*	When $\log_a M = x, a^x = M$ Taking the logarithm, to the base b, of both sides, $\log_b a^x = \log_b M$ $x\log_b a = \log_b M$ $x = \frac{\log_b M}{\log_b a}$ Therefore, $$\log_a M = \frac{\log_b M}{\log_b a}$$ (where $a > 0, b > 0, M > 0$ and $a \neq 1, b \neq 1$)	L11a
	$\log_a b = \frac{1}{\log_b a} \qquad \log_a b \cdot \log_b a = 1$	L11b L13a
	$a^{\log_a M} = M$	L14b
	When $a > 0$ and $a \neq 1$, a function of the form $y = \log_a x$ is called the *logarithmic function of x*, where a is the base. When $0 < a < 1$  The graph of the logarithmic function $y = \log_a x$ passes through point $(1, 0)$, and its asymptote is the y-axis. When $1 < a$  	L16b
Translations	The graph of the logarithmic function $y = \log_a(x - p) + q$ (where $a > 0$ and $a \neq 1$) is a translation of the graph of $y = \log_a x$, p units along the x-axis, and q units along the y-axis.	L17b

LEVEL L

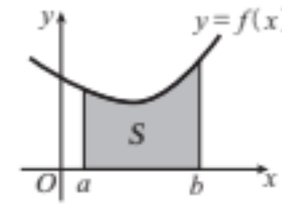
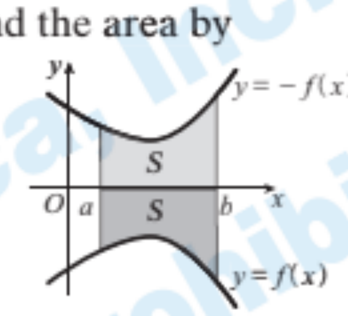
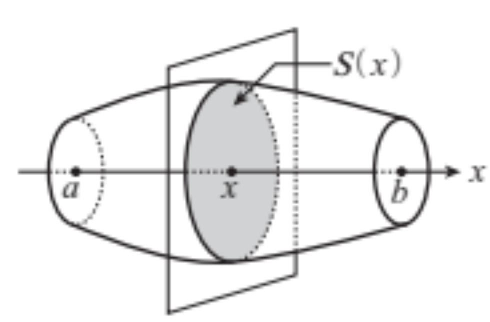
Topic	Formulas & Notes	Reference
	Given the logarithmic functions $y = \log_a p$, $y = \log_a q$, When $a > 1$, then $p < q \Leftrightarrow \log_a p < \log_a q$. When $0 < a < 1$, then $p < q \Leftrightarrow \log_a p > \log_a q$.	L19b
Logarithmic Equations and Inequalities *Solving Methods*	• If the initial equation can be written in the form $\log_a A = \log_a B$, we can equate the antilogarithms, $A = B$. • If we let $\log_a x = X$ and can rewrite the equation in terms of X, then we solve for X. When $\log_a x = b$, then $x = a^b$. • We check all solutions by considering that: antilogarithms are > 0, the base is > 0, and the base $\neq 1$	L22 L23
	When $a > 1$, $\log_a x > b \rightleftarrows x > a^b$ When $0 < a < 1$, $\log_a x > b \rightleftarrows 0 < x < a^b$	L24
Modulus Functions	$\lvert a \rvert$ is called the *absolute value* of a. When a is positive or zero, $\lvert a \rvert = a$. When a is negative, $\lvert a \rvert = -a$.	L31a
Limits and Derivatives	The *average rate of change* of $y = f(x)$ when x varies from $x = a$ to $x = b$ is expressed as: $\dfrac{f(b) - f(a)}{b - a}$	L43a
Differential Coefficient	Given $y = f(x)$, the average rate of change, when x varies from a to $a + h$, can be expressed as $\dfrac{f(a+h) - f(a)}{h}$. In this case, $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$  $f'(a)$ is the *differential coefficient* for $x = a$. ($f'(a)$ indicates the slope of the line that is tangent to the graph of $y = f(x)$ at $x = a$.) 	L44a
Derivative of $f(x)$	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$	L46a
	When n is a natural number, $(x^n)' = nx^{n-1}$.	L47b
Rules of Differentiation	If $y = k$, then $y' = 0$ (where k is a constant) If $y = kf(x)$, then $y' = kf'(x)$ (where k is a constant) If $y = f(x) + g(x)$, then $y' = f'(x) + g'(x)$ If $y = f(x) - g(x)$, then $y' = f'(x) - g'(x)$ If $y = f(x)g(x)$, then $y' = f'(x)g(x) + f(x)g'(x)$	L48a L49b
Tangent Lines	The slope of the tangent line at point $(a, f(a))$, on the curve $y = f(x)$, is $f'(a)$. The equation of the tangent line at point $(a, f(a))$ is as follows: $y - f(a) = f'(a)(x - a)$ 	L51a

Topic	Formulas & Notes	Reference
Relative Maxima and Minima 1	 The cubic function $f(x) = x^3 - 9x^2 + 24x - 15$ is: • increasing when $x < 2$ • decreasing when $2 < x < 4$ • increasing when $4 < x$ And, • At $x = 2$ the function has a relative maximum. • At $x = 4$ the function has a relative minimum.	L61a
	We do not identify that a function has a relative maximum or a relative minimum until we find out the variation around the point where $y' = 0$ (or $f'(x) = 0$). 	L62b
Variation Table	•When $y' > 0$ we use $+$, when $y' < 0$ we use $-$ • ↗ means increasing, ↘ means decreasing •There is a relative maximum value when the arrow changes from ↗ to ↘, and there is a relative minimum value when the arrow changes from ↘ to ↗.	L62b
	For all values of x, When $y' > 0$, y is always increasing, and thus there is no relative extreme value. When $y' < 0$, y is always decreasing, and thus there is no relative extreme value.	L68a
Relative Maxima and Minima 2	When the coefficient of x^3 is a letter, as in $ax^3 + 2ax^2 + ax + 1$, we find the relative extreme values by analyzing 2 cases (a) when $a > 0$ and (b) when $a < 0$.	L73b
Maxima and Minima 1	To find a maximum and a minimum value of a function $y = f(x)$ on an interval $a \leq x \leq b$, (a) We create a variation table on the interval $a \leq x \leq b$. (b) We determine and then compare the relative extreme values and the values of $f(a)$ and $f(b)$ on both ends of the interval.	L82b
	In exercises where we have to find the minimum value, when there is no relative minimum value in the interval, we obtain the minimum value either at the right end or the left end of the domain. In the exercises where we have to find the maximum value, when there is no relative maximum value in the interval, we obtain the maximum value either at the right end or the left end of the domain.	L83b L84b

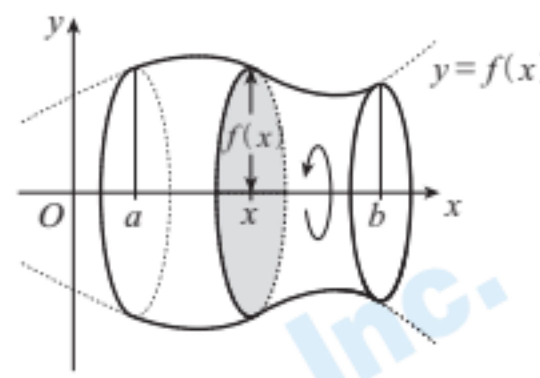
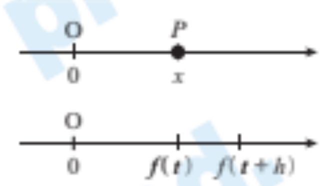
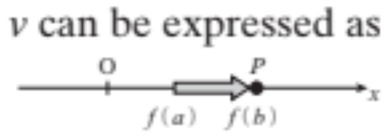
LEVEL L

Topic	Formulas & Notes	Reference
Applications to Equations and Inequalities	Real roots of an equation $f(x) = 0$ will be the x-coordinates of common points where the graph of the function $y = f(x)$ and the x-axis cross.	L101a
Indefinite and Definite Integrals	When one of the indefinite integrals of $f(x)$ is $F(x)$, $\int f(x)\,dx = F(x) + C$ (C is the constant of integration.)	L111b
Formula for Indefinite Integrals	When n is a positive integer, or zero, $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ (C is the constant of integration.)	L111b
Properties of Indefinite Integrals	$\int kf(x)\,dx = k\int f(x)\,dx$ (k is a constant.) $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$ $\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$	L112a
Definition of Definite Integral	When one of the indefinite integrals of $f(x)$ is $F(x)$, $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$	L117b
Definite Integrals 1 *Properties of Definite Integrals*	$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ (k is a constant.) $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$ $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)dx - \int_a^b g(x)\,dx$	L121a
Property of Definite Integrals	$\int_{-a}^{a} x^n\,dx = \begin{cases} 0 & \text{(when } n \text{ is } 1, 3, 5, \ldots) \\ 2\int_0^a x^n\,dx & \text{(when } n \text{ is } 0, 2, 4, \ldots) \end{cases}$	L123b
Property of Definite Integrals	When $\alpha < \beta$, $\int_\alpha^\beta (x - \alpha)(x - \beta)\,dx = -\frac{1}{6}(\beta - \alpha)^3$	L124b
Properties of Definite Integrals	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$	L126b

LEVEL L

Topic	Formulas & Notes	Reference
Definite Integrals 2	$\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = f(x)$ (where a is a constant)	L131b
Areas 1	Generally when $f(x) \geq 0$ we can find the area, S, of the region enclosed by the curve $y = f(x)$ and the x-axis in the interval of $[a, b]$ by using the following formula: $$S = \int_a^b f(x)\,dx$$ 	L141b
	In the interval $[a, b]$ there are two regions, one enclosed by $y = f(x)$ and the x-axis and one enclosed by $y = -f(x)$ and the x-axis, that are symmetric with respect to the x-axis and equal in area. Therefore, when $f(x) \leq 0$ we can find the area by using the following formula: $$S = -\int_a^b f(x)\,dx$$ 	L142a
	The area, S, of the region enclosed by two curves, $y = f(x)$ and $y = g(x)$, where $f(x) \geq g(x)$ in the interval $[a, b]$ is: $$S = \int_a^b [f(x) - g(x)]\,dx \quad (\text{where } f(x) \geq g(x))$$	L146a
Area of the Region Involving a Parabola	$\displaystyle\int_\alpha^\beta (x-\alpha)(x-\beta)\,dx = -\frac{1}{6}(\beta-\alpha)^3$	L148a
Volumes	If $S(x)$ is a function of x representing the cross-sectional area of the region on a given solid cut by a plane which is perpendicular to the x-axis, then the volume for values where $a \leq x \leq b$ is given by the following formula: $$V = \int_a^b S(x)\,dx$$ 	L161a

[NOTES]

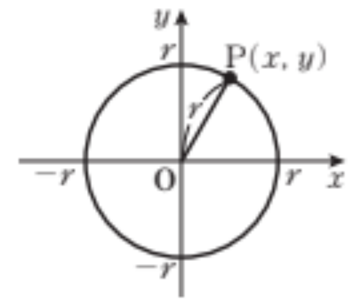
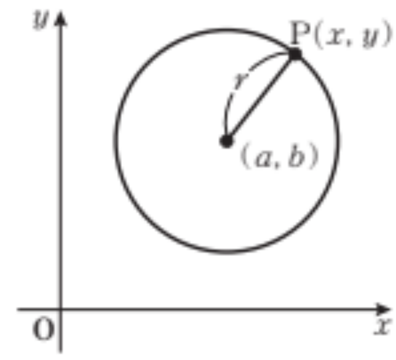
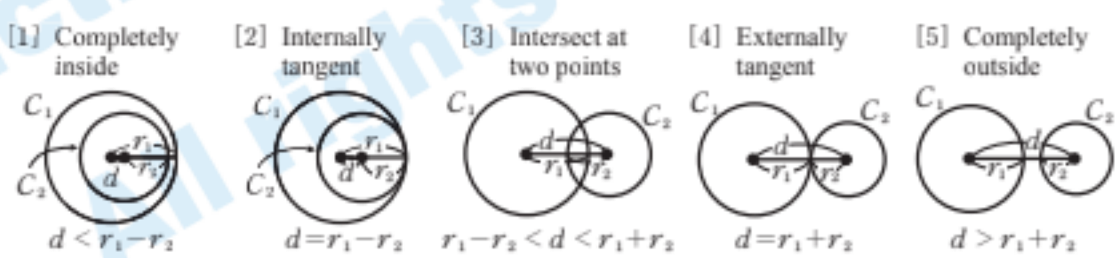
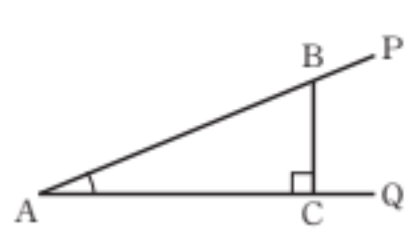
Topic	Formulas & Notes	Reference
Volume of a Solid of Revolution	The solid that forms by rotating a line or curve around an axis is called a *solid of revolution*. We let V be the volume of the solid that forms by rotating the region enclosed by the function $y = f(x)$, the x-axis, line $x = a$, and line $x = b$. Since the surface, which is cut by a plane perpendicular to the x-axis, would be a circle, its area is $S(x) = \pi y^2 = \pi[f(x)]^2$ Therefore, the volume, V, of the solid of revolution for values where $a \leq x \leq b$ is: $V = \int_a^b \pi y^2\,dx = \pi \int_a^b [f(x)]^2\,dx$ 	L165a
Velocity and Distance	When a point P moves on a number line, if the position of the point P at time t is expressed as the x-coordinate, then x is a function of t. Letting this function be $x = f(t)$, the *average velocity* of point P from time t to time $t + h$ is expressed as:  $\dfrac{f(t+h) - f(t)}{h}$	L171a
Velocity	When the position at time t, of point P moving on a number line, is expressed as $x = f(t)$, we can express the *velocity* of point P as: $v = \dfrac{dx}{dt} = \lim_{h \to 0} \dfrac{f(t+h) - f(t)}{h} = f'(t)$	L171b
Displacement	Given a point P moving on a number line, if we let the position at time t be $x = f(t)$, the velocity v can be expressed as $v = \dfrac{dx}{dt} = f'(t)$.  The *displacement* (the change in position) of point P from $t = a$ to $t = b$ is expressed as follows: $f(b) - f(a) = \int_a^b f'(t)dt = \int_a^b v\,dt$	L175a
Distance	Generally, given a point P moving at time t with velocity v, the distance, s, traveled from time $t = a$ to $t = b$ is expressed by the following formula: $s = \int_a^b \lvert v \rvert\,dt$	L176b

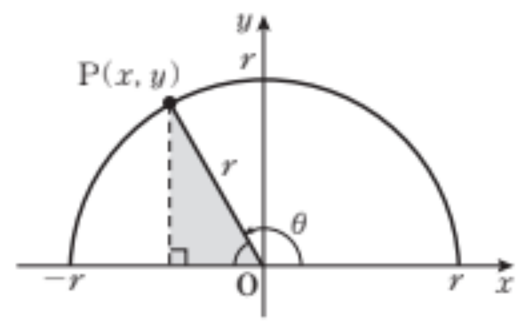
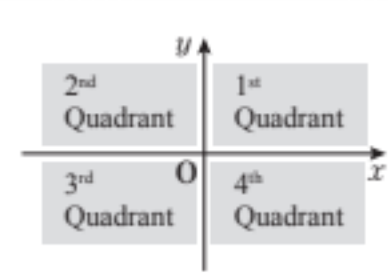
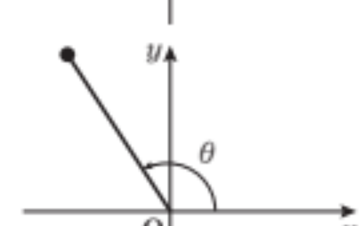
LEVEL M

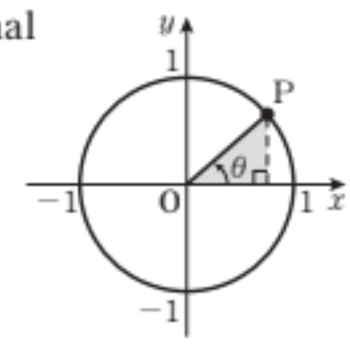
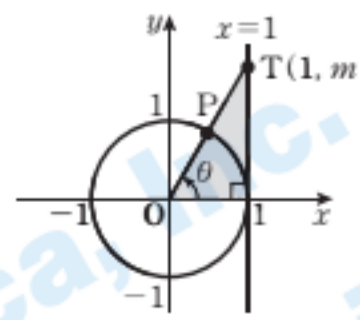
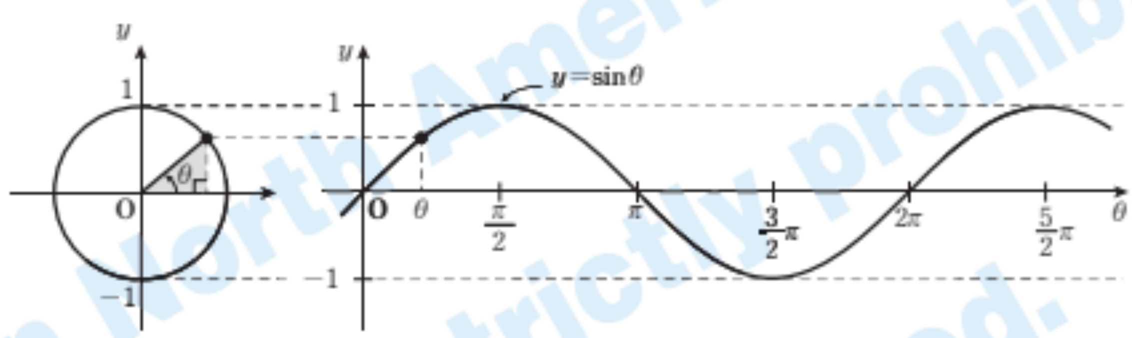
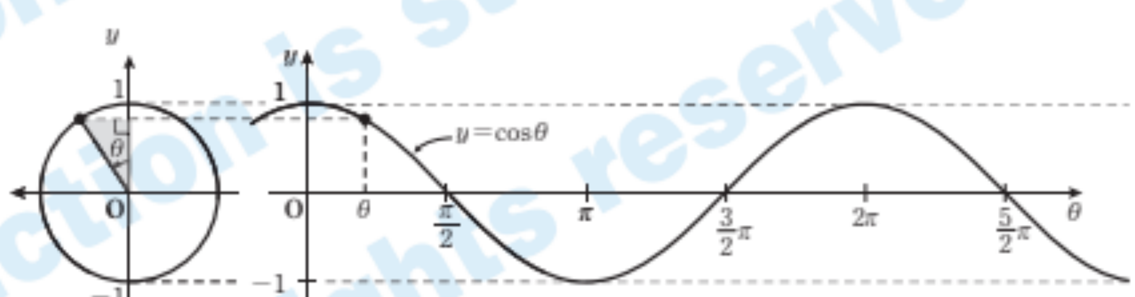
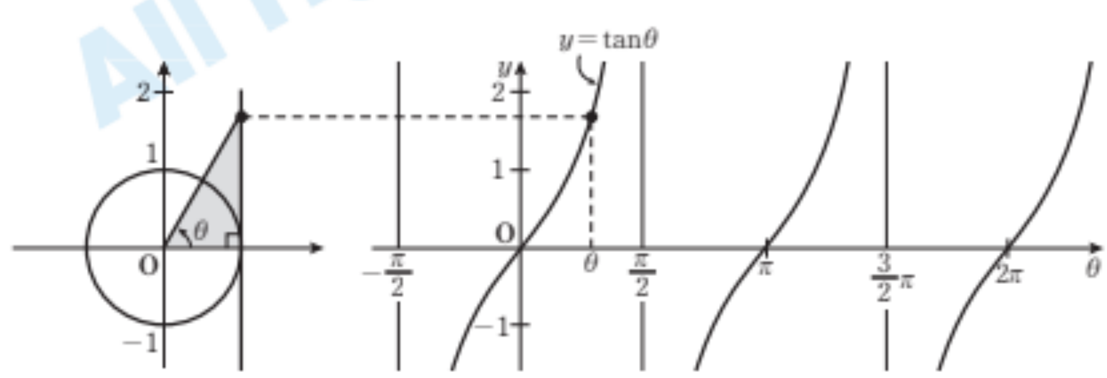
Topic	Formulas & Notes	Reference
Points and Lines 1 *Distance Formula*	The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Also, the distance between origin O and point $A(x_1, y_1)$ is $OA=\sqrt{x_1{}^2+y_1{}^2}$	M1b
Coordinates of Internal/External Dividing Points	Given points $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the points that divide line segment AB in the ratio $m : n$ are as follows: Internally, $\left(\frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right)$ Externally, $\left(\frac{-nx_1+mx_2}{m-n}, \frac{-ny_1+my_2}{m-n}\right)$	M6b
Midpoint	The coordinates of the midpoint M of line segment AB, where $A(x_1, y_1)$ and $B(x_2, y_2)$ are given by $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	M7a
Center of Gravity of Triangles	Given $\triangle ABC$ with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, the coordinates of the center of gravity G are given by $G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$	M8b
Points and Lines 2 *Equation of a Line I*	The equation of a line passing through point and (x_1, y_1) with slope m is $y-y_1=m(x-x_1)$	M11a
Parallel Condition	For two lines $y=m_1x+n_1$ and $y=m_2x+n_2$, the parallel condition is $m_1=m_2$.	M14a
Perpendicular Condition	For two lines $y=m_1x+n_1$ and $y=m_2x+n_2$, the perpendicular condition is $m_1m_2=-1$.	M15b
Points and Lines 3 *Distance from a Point to a Line*	The distance d from point (x_1, y_1) to line $ax+by+c=0$ is $d=\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$	M25b

[NOTES]

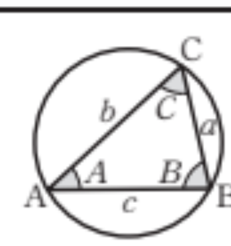
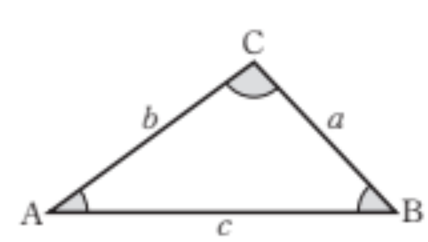
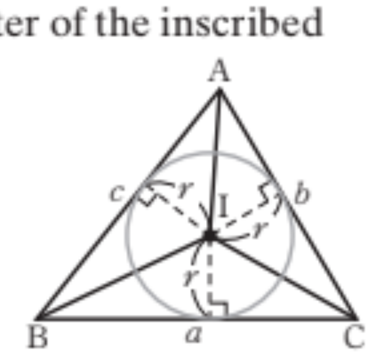
LEVEL M

Topic	Formulas & Notes	Reference
Circles 1 *Equation of a Circle I*	The equation of a circle with center at origin O and radius r is $x^2+y^2=r^2$ 	M31a
Equation of a Circle II	The equation of a circle with center at point (a, b) and radius r is $(x-a)^2+(y-b)^2=r^2$ 	M31b
Positional Relationship between a Line and a Circle	When the quadratic equation $ax^2+bx+c=0$ is obtained after y is eliminated from each equation of a line and a circle, let the discriminant be $D(=b^2-4ac)$. $D>0$ ⇔ They intersect at two different points. $D=0$ ⇔ They are tangent and intersect at only one point. $D<0$ ⇔ They do not intersect.	M37a
Circles 2 *Tangent to a Circle*	The equation of the tangent to circle $x^2+y^2=r^2$ at point $P(x_1, y_1)$ on the circle is $x_1x+y_1y=r^2$	M41b
Positional Relationship between Two Circles	Let the radii of two circles C_1 and C_2 be r_1 and r_2 $(r_1>r_2)$ respectively and the distance between the centers of the circles be d.  ※ [3]~[5] are also true when $r_1=r_2$. If it is not possible to define which is greater between the values of r_1 and r_2, use $\lvert r_1-r_2\rvert$ instead of r_1-r_2 in [1]~[3].	M49a
Trigonometric Ratios 1	Given ∠PAQ, drop a perpendicular BC from point B on AP to AQ and form a right-angled triangle ABC. When A represents the size of ∠A,  $\frac{BC}{AB}$ is called ***the sine of A*** and expressed as $\sin A$. $\frac{AC}{AB}$ is called ***the cosine of A*** and expressed as $\cos A$. $\frac{BC}{AC}$ is called ***the tangent of A*** and expressed as $\tan A$.	M81a

Topic	Formulas & Notes	Reference
Trigonometric Identities I	$\tan A = \dfrac{\sin A}{\cos A}$ $\sin^2 A + \cos^2 A = 1$	M86a
Trigonometric Identity II	$1 + \tan^2 A = \dfrac{1}{\cos^2 A}$	M87b
Trigonometric Ratios 2 *Trigonometric Ratios of $0° \leq \theta \leq 180°$*	$\sin\theta = \dfrac{y}{r}$ $\cos\theta = \dfrac{x}{r}$ $\tan\theta = \dfrac{y}{x}$ 	M91b
Trigonometric Ratios of $90° - \theta$	$\sin(90° - \theta) = \cos\theta$ $\cos(90° - \theta) = \sin\theta$ $\tan(90° - \theta) = \dfrac{1}{\tan\theta}$	M98a
Trigonometric Ratios of $180° - \theta$	$\sin(180° - \theta) = \sin\theta$ $\cos(180° - \theta) = -\cos\theta$ $\tan(180° - \theta) = -\tan\theta$	M98b
Properties of Trigonometric Functions 1 *Definitions of Trigonometric Functions*	$\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$, $\tan\theta = \dfrac{y}{x}$	M101a
Trigonometric Functions of $\theta + 360° \times n$	$\sin(\theta + 360° \times n) = \sin\theta$ $\cos(\theta + 360° \times n) = \cos\theta$ $\tan(\theta + 360° \times n) = \tan\theta$	M102b
Trigonometric Functions of $-\theta$	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$	M104b
Trigonometric Functions of $\theta + 180°$	$\sin(\theta + 180°) = -\sin\theta$ $\cos(\theta + 180°) = -\cos\theta$ $\tan(\theta + 180°) = \tan\theta$	M105b
Trigonometric Functions of $\theta + 90°$	$\sin(\theta + 90°) = \cos\theta$ $\cos(\theta + 90°) = -\sin\theta$ $\tan(\theta + 90°) = -\dfrac{1}{\tan\theta}$	M106b
	The four regions on a coordinate plane are called the ***1st Quadrant***, ***2nd Quadrant***, ***3rd Quadrant*** and ***4th Quadrant***.  When the terminal side of θ is in the 2nd Quadrant, θ is called an ***angle in the 2nd Quadrant***. 	M107a

Topic	Formulas & Notes	Reference
Properties of Trigonometric Functions 2 *Circular Measure*	Since $180° = \pi$ radians, $1° = \dfrac{\pi}{180}$ radians, $\quad$ 1 radian $= \dfrac{180°}{\pi}$	M111a
Graphs of Trigonometric Functions	Let the point of intersection of the terminal side of θ and the unit circle be P. The y-coordinate of P is $\sin\theta$. The x-coordinate of P is $\cos\theta$. Using these, we can draw the graphs of $y = \sin\theta$ and $y = \cos\theta$.  Let the point of intersection of line OP and line $x = 1$ be $\mathrm{T}(1, m)$. Then $\tan\theta = m$ is true. Using this, we can draw the graph of $y = \tan\theta$. 	M131a M132a M137a
Graph of y = sinθ		M131a
Graph of y = cosθ		M132a
Graph of y = tanθ		M137a
Addition Formulas 1 *Addition Formulas I*	$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	M152a
Addition Formulas II	$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$	M155b

LEVEL M

Topic	Formulas & Notes	Reference
Addition Formulas 2 *Double-Angle Formulas*	$\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$ $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$	M161b
Triple-Angle Formulas	$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$ $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$	M163b
Half-Angle Formulas	$\sin^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{2}$, $\cos^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{2}$, $\tan^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{1+\cos\alpha}$	M164b
Addition Formulas 3 *Conversion of asinθ + bcosθ*	$a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}\sin(\theta+\alpha)$ where $\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$	M171b
Product-to-Sum Formulas	$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$ $\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$ $\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$ $\sin\alpha\sin\beta = -\dfrac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$	M175b
Sum-to-Product Formulas	$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$	M177b
Laws of Sines and Cosines *Law of Sines*	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$ (R is the radius of the circumscribed circle of △ABC) 	M182a
Law of Cosines	Given △ABC, $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = c^2 + a^2 - 2ca\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$ 	M184b
Area of Triangles *Formula*	$S = \dfrac{1}{2}bc\sin A = \dfrac{1}{2}ca\sin B = \dfrac{1}{2}ab\sin C$	M191a
Area of a Triangle with an Inscribed Circle	Let S be the area of △ABC, I be the center of the inscribed circle, and r be the radius. $S = \triangle\mathrm{IBC} + \triangle\mathrm{ICA} + \triangle\mathrm{IAB}$ $= \dfrac{1}{2}ar + \dfrac{1}{2}br + \dfrac{1}{2}cr$ $= \dfrac{1}{2}r(a+b+c)$ 	M196a

[NOTES]

LEVEL N

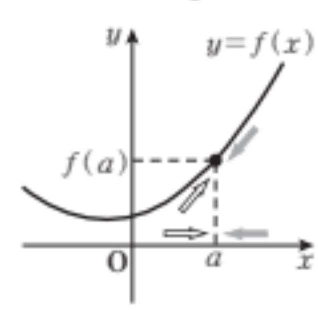
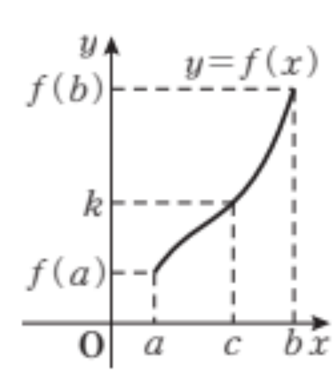
Topic	Formulas & Notes	Reference
Arithmetic Sequences *General Term of an Arithmetic Sequence*	A sequence whose terms are found by successively adding a fixed number d to the 1st term a is called an ***arithmetic sequence*** (or ***arithmetic progression***). d is called the ***common difference*** of the arithmetic sequence. The general term of an arithmetic sequence $\{a_n\}$ with 1st term a and common difference d is $a_n=a+(n-1)d$	N2a N2b
Sum of an Arithmetic Sequence	Let S_n be the sum of an arithmetic sequence with 1st term a, common difference d, last term l and number of terms n. $S_n=\frac{1}{2}n(a+l)=\frac{1}{2}n[2a+(n-1)d]$	N6b
Geometric Sequences *General Term of a Geometric Sequence*	A sequence whose terms are found by successively multiplying a fixed number r to the 1st term a is called a ***geometric sequence*** (or ***geometric progression***). r is called the ***common ratio*** of the geometric sequence. The general term of a geometric sequence $\{a_n\}$ with 1st term a and common ratio r is $a_n=ar^{n-1}$	N11a N12a
Sum of a Geometric Sequence	Let S_n be the sum of a geometric sequence with 1st term a, common ratio r and number of terms n. When $r\neq1$, $S_n=\frac{a(1-r^n)}{1-r}=\frac{a(r^n-1)}{r-1}$ When $r=1$, $S_n=na$ ← When $r=1$, S_n is the sum of n terms of a.	N16b
Various Sequences 1 *Summation Formulas I*	$\sum_{k=1}^{n}k=\frac{1}{2}n(n+1)$, $\sum_{k=1}^{n}c=nc$ (c is a constant)	N22a
Summation Properties	$\sum_{k=1}^{n}(a_k+b_k)=\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k$, $\sum_{k=1}^{n}ca_k=c\sum_{k=1}^{n}a_k$ (c is a constant)	N22b
Summation Formula II	$\sum_{k=1}^{n}k^2=\frac{1}{6}n(n+1)(2n+1)$	N23b
Summation Formula III	$\sum_{k=1}^{n}k^3=\left[\frac{1}{2}n(n+1)\right]^2$	N24b
Summation Formula IV	$\sum_{k=1}^{n}ar^{k-1}=\frac{a(1-r^n)}{1-r}=\frac{a(r^n-1)}{r-1}$ $(r\neq1)$	N25a
Various Sequences 2	The term which is derived by taking the difference between two consecutive terms in the sequence $\{a_n\}$ is $b_n=a_{n+1}-a_n$ $(n=1,\ 2,\ 3,\ \ldots)$ ← $a_1\ a_2\ a_3\ a_4\ \cdots\ a_n\ a_{n+1}\ \cdots$ / $b_1\ b_2\ b_3\ \cdots\ b_n\ \cdots$ Generally, the sequence $\{b_n\}$ is called the ***sequence of differences*** of $\{a_n\}$.	N31b

LEVEL N

Topic	Formulas & Notes	Reference
Sequence of Differences and General Term	Let $\{b_n\}$ be the sequence of differences of the sequence $\{a_n\}$. When $n \geqslant 2$, $a_n = a_1 + \sum_{k=1}^{n-1} b_k$	N32a
Sum of a Sequence and General Term	Let S_n be the sum of the first n terms of the sequence $\{a_n\}$. The 1st term a_1 is $a_1 = S_1$. When $n \geqslant 2$, $a_n = S_n - S_{n-1}$	N34a
Recurrence Relations	Given that the sequence $\{a_n\}$ satisfies the following two conditions: (i) $a_1 = 2$ (ii) $a_{n+1} = 3a_n - 1 \quad (n = 1, 2, 3, \ldots)$ When each term is successively determined from $a_2, a_3, a_4, \ldots$ one after another, then one sequence $\{a_n\}$ will be defined. A condition such as (ii), where the expression defines the relationship between two or more consecutive terms in a sequence, is called a ***recurrence relation***.	N41a
Mathematical Induction	To prove that a proposition P is true for all natural numbers n by mathematical induction, the following two statements have to be proved. (i) P is true when $n = 1$. (ii) If P is true when $n = k$, then P is also true when $n = k + 1$.	N51a
Infinite Sequences	A sequence with infinite terms $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called an ***infinite sequence***, and it is expressed as $\{a_n\}$.	N61a
	When $\{a_n\}$ ***diverges to positive (or negative) infinity***, the limit is positive (or negative) infinity, and is expressed as $\lim_{n\to\infty} a_n = +\infty \qquad \lim_{n\to\infty} a_n = -\infty$ Divergent sequences which do not diverge to positive or negative infinity are said to ***oscillate***.	N62a
Limit of Sequence	Converges: $\lim_{n\to\infty} a_n = \alpha$ (converges to a constant value α) ⋯① Diverges: $\lim_{n\to\infty} a_n = \infty$ (diverges to positive infinity) ⋯② $\lim_{n\to\infty} a_n = -\infty$ (diverges to negative infinity) ⋯③ Oscillates (no limit) ⋯④	N62b
Properties of Limits of Sequences	When the sequences $\{a_n\}$ and $\{b_n\}$ converge, where $\lim_{n\to\infty} a_n = \alpha$ and $\lim_{n\to\infty} b_n = \beta$, $\lim_{n\to\infty} ka_n = k\alpha$ (k is a constant) $\lim_{n\to\infty}(a_n + b_n) = \alpha + \beta$, $\lim_{n\to\infty}(a_n - b_n) = \alpha - \beta$ $\lim_{n\to\infty} a_n b_n = \alpha\beta$ $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\alpha}{\beta}$ $(\beta \neq 0)$	N63a

Topic	**Formulas & Notes**	**Reference**
Limits of Sequences and Their Relationships	1. For all n, when $a_n \leqslant b_n$, if $\lim\limits_{n\to\infty} a_n=\alpha$ and $\lim\limits_{n\to\infty} b_n=\beta$, then $\alpha \leqslant \beta$ if $\lim\limits_{n\to\infty} a_n=\infty$, then $\lim\limits_{n\to\infty} b_n=\infty$ 2. For all n, when $a_n \leqslant c_n \leqslant b_n$, if $\lim\limits_{n\to\infty} a_n=\lim\limits_{n\to\infty} b_n=\alpha$, then $\lim\limits_{n\to\infty} c_n=\alpha$	N67a
Infinite Geometric Sequences *Limit of an Infinite Geometric Sequence* $\{r^n\}$	When $r>1$, $\lim\limits_{n\to\infty} r^n=\infty$ ⋯ Diverges When $r=1$, $\lim\limits_{n\to\infty} r^n=1$ When $\lvert r\rvert<1$, $\lim\limits_{n\to\infty} r^n=0$ } ⋯ Converges When $r \leqslant -1$, Oscillates (no limit) ⋯ Diverges	N71b
Infinite Geometric Series	Given an infinite sequence $\{a_n\}$, the expression $a_1+a_2+a_3+\cdots+a_n+\cdots$ ⋯① is called the ***infinite series***, where a_1 and a_n are called the ***1st term*** and the ***nth term*** respectively. Let S_n be the sum of the first n terms. When the infinite sequence $\{S_n\}$ converges, the infinite series ① is said to converge. When the infinite sequence $\{S_n\}$ diverges, the infinite series ① is said to diverge. Likewise, $a+ar+ar^2+\cdots+ar^{n-1}+\cdots$ which is the infinite series that is derived from the infinite geometric sequence with 1st term a and common ratio r is called the ***infinite geometric series*** with 1st term a and common ratio r.	N81a
Convergence and Divergence of an Infinite Geometric Series	Given an infinite geometric series $a+ar+ar^2+\cdots+ar^{n-1}+\cdots$, the following is true. When $a \neq 0$, if $\lvert r\rvert<1$, then the series converges and the sum is $\frac{a}{1-r}$; if $\lvert r\rvert \geqslant 1$, then the series diverges. When $a=0$, the series converges and the sum is 0.	N81b
Infinite Series	Given the infinite series $a_1+a_2+a_3+\cdots+a_n+\cdots$ ⋯①, the sum of the first n terms $S_n=a_1+a_2+a_3+\cdots+a_n$ is called the ***partial sum*** of the first n terms of the infinite series. The infinite series ① can be written as $\sum\limits_{n=1}^{\infty} a_n$.	N91a
Properties of Infinite Series	When the infinite series $\sum\limits_{n=1}^{\infty} a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ converge, where $\sum\limits_{n=1}^{\infty} a_n=S$ and $\sum\limits_{n=1}^{\infty} b_n=T$, the following properties are true: $\sum\limits_{n=1}^{\infty} ka_n=kS$ (k is a constant) $\sum\limits_{n=1}^{\infty}(a_n+b_n)=S+T$ $\sum\limits_{n=1}^{\infty}(a_n-b_n)=S-T$	N95a

LEVEL N

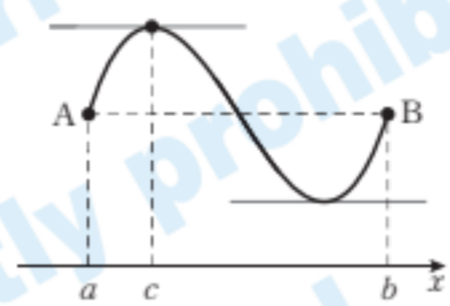
Topic	Formulas & Notes	Reference
Convergence and Divergence of Infinite Series	If the infinite series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n=0$. If the sequence $\{a_n\}$ does not converge to 0, then the infinite series $\sum_{n=1}^{\infty} a_n$ diverges.	N96a
Limits of Functions 1 *Properties of Limits and Functions*	If $\lim_{x\to a} f(x)=\alpha$ and $\lim_{x\to a} g(x)=\beta$, then $\lim_{x\to a} kf(x)=k\alpha$ (k is a constant) $\lim_{x\to a}[f(x)+g(x)]=\alpha+\beta$, $\lim_{x\to a}[f(x)-g(x)]=\alpha-\beta$ $\lim_{x\to a} f(x)g(x)=\alpha\beta$ $\lim_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\alpha}{\beta}$ $(\beta\neq 0)$	N101a
Existence of a Limit	If $\lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)=\alpha$, then $\lim_{x\to a} f(x)=\alpha$ exists. If $\lim_{x\to a^+} f(x)\neq\lim_{x\to a^-} f(x)$, then $\lim_{x\to a} f(x)$ does not exist.	N105a
	Given $\lim_{x\to a}\dfrac{f(x)}{g(x)}=\alpha$, when $\lim_{x\to a} g(x)=0$, then $\lim_{x\to a} f(x)=0$.	N108b
Limits of Trigonometric Functions *Limit of* $\dfrac{\sin x}{x}$	$\lim_{x\to 0}\dfrac{\sin x}{x}=1$	N121b
Continuous and Discontinuous Functions	Generally, the function $f(x)$ is said to be ***continuous*** at $x=a$ if $f(x)$ satisfies the following two conditions with respect to a, which is the value of x within the domain. (i) $\lim_{x\to a} f(x)$ exists. (ii) $\lim_{x\to a} f(x)=f(a)$ is true. 	N131a
Intermediate Value Theorem	The interval $a\leqq x\leqq b$ is called a ***closed interval*** and the interval $a<x<b$ is called an ***open interval***. They are expressed as $[a, b]$ and (a, b), respectively. Generally, the Intermediate Value Theorem is explained as follows: If the function $f(x)$ is continuous on the closed interval $[a, b]$ and $f(a)\neq f(b)$, then there is at least one value c that satisfies $f(c)=k$ and $a<c<b$ for any arbitrary value k which lies between $f(a)$ and $f(b)$. 	N137a
Differentiation 1	Given the function $f(x)$, if the limit value $\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ exists, then it is called the ***derivative value*** of $f(x)$ at $x=a$ and expressed as $f'(a)$. In this case, $f(x)$ is said to be ***differentiable*** at $x=a$.	N141a

LEVEL N

Topic	Formulas & Notes	Reference
Properties of Derivatives	When k is a constant and n is a positive integer, if $y=x^n$, then $y'=nx^{n-1}$ if $y=kf(x)$, then $y'=kf'(x)$ if $y=f(x)+g(x)$, then $y'=f'(x)+g'(x)$ if $y=f(x)-g(x)$, then $y'=f'(x)-g'(x)$	N143a
Product Rule	$[f(x)g(x)]'=f'(x)g(x)+f(x)g'(x)$	N144a
Quotient Rule	$\left[\frac{f(x)}{g(x)}\right]'=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$, $\left[\frac{1}{g(x)}\right]'=-\frac{g'(x)}{[g(x)]^2}$	N146b
Derivative of x^n	When n is an integer, $(x^n)'=nx^{n-1}$	N148a
Differentiation 2 *Chain Rule I*	$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$	N151b
Chain Rule II	$[f(g(x))]'=f'(g(x))g'(x)$	N152a
Differentiation Formula for Inverse Functions	$\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$ $\left(\frac{dx}{dy}\neq 0\right)$	N155a
Derivative of x^p	When p is a rational number, $(x^p)'=px^{p-1}$	N156a
Differentiation of Trigonometric Functions *Derivatives of Trigonometric Functions*	$(\sin x)'=\cos x$, $(\cos x)'=-\sin x$, $(\tan x)'=\frac{1}{\cos^2 x}$	N162a
Differentiation of Logarithmic and Exponential Functions	When examining the value of $(1+k)^{\frac{1}{k}}$ by substituting k with a value which is close to 0, it approaches a constant value as shown below. e is an irrational number and $e=2.7182818\ldots$.	N171a
Derivatives of Logarithmic Functions I	$(\ln x)'=\frac{1}{x}$, $(\log_a x)'=\frac{1}{x\ln a}$	N171b
Derivatives of Logarithmic Functions II	$(\ln\|x\|)'=\frac{1}{x}$, $(\log_a\|x\|)'=\frac{1}{x\ln a}$	N172b

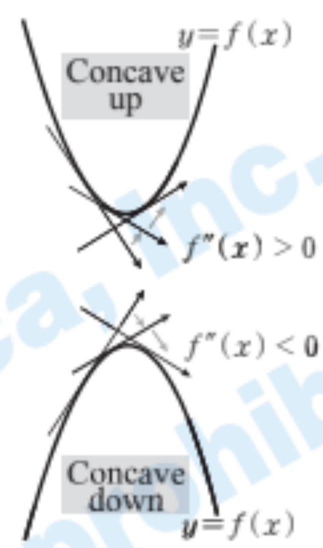
k	$(1+k)^{\frac{1}{k}}$	k	$(1+k)^{\frac{1}{k}}$
0.1	2.59374…	−0.1	2.86797…
0.01	2.70481…	−0.01	2.73199…
0.001	2.71692…	−0.001	2.71964…
0.0001	2.71814…	−0.0001	2.71841…
0.00001	2.71826…	−0.00001	2.71829…

LEVEL N

Topic	Formulas & Notes	Reference
Derivatives of Exponential Functions	$(e^x)'=e^x$, $(a^x)'=a^x\ln a$	N177a
Differentiation of Various Functions and Higher Order Derivatives *Derivative of Functions Represented by a Parameter*	When $x=f(t)$ and $y=g(t)$, $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{g'(t)}{f'(t)}$	N183a
	In general, the function obtained by differentiating the fuction $y=f(x)$ n times is called the ***n^{th} order derivative*** of $f(x)$ and can be written as $y^{(n)}$, $f^{(n)}(x)$, $\dfrac{d^ny}{dx^n}$, or $\dfrac{d^n}{dx^n}f(x)$	N185a N189a
Various Properties of Derivatives *Rolle's Theorem*	If the function $f(x)$ is continuous on the closed interval $[a, b]$, differentiable on the open interval (a, b) and $f(a)=f(b)$, then there exists at least one value c such that $f'(c)=0$ and $a<c<b$. 	N196a
Mean Value Theorem	If the function $f(x)$ is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists at least one value c such that $\dfrac{f(b)-f(a)}{b-a}=f'(c)$ and $a<c<b$.	N197a

[NOTES]

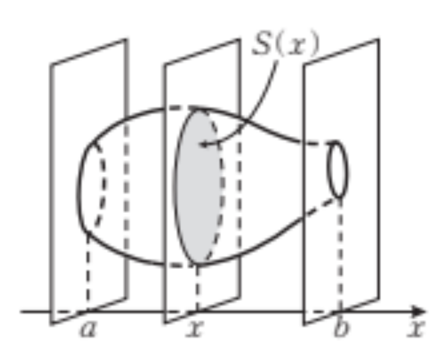
LEVEL O

Topic	Formulas & Notes	Reference
Tangents and Normals *Equation of a Tangent Line*	The equation of the tangent to the curve $y=f(x)$ at point $(a, f(a))$ is $y-f(a)=f'(a)(x-a)$	O1a
Equation of a Normal Line	The equation of the normal to the curve $y=f(x)$ at point $(a, f(a))$ is $y-f(a)=-\frac{1}{f'(a)}(x-a)$ when $f'(a)\neq 0$ $x=a$ when $f'(a)=0$	O3a
Concavity of Curves *Concave Up* *Concave Down*	When a function $f(x)$ has the second order derivative $f''(x)$, in the interval where $f''(x)>0$, the curve $y=f(x)$ is concave up; and in the interval where $f''(x)<0$, the curve $y=f(x)$ is concave down. 	O21a
Various Applications of Differentiation *Linear Approximation I*	When the function $f(x)$ is differentiable at $x=a$ and the value of $\lvert h\rvert$ is approaching 0, $f(a+h)\approx f(a)+f'(a)h$	O48a
Linear Approximation II	When the value of $\lvert x\rvert$ is approaching 0, $f(x)\approx f(0)+f'(0)x$	O48b
Indefinite Integrals 1	The function which becomes $f(x)$ when differentiated is called the ***indefinite integral*** or ***antiderivative*** of $f(x)$ and is expressed as $\int f(x)dx$. Let an indefinite integral of $f(x)$ be $F(x)$. Then, it is expressed as $\int f(x)dx=F(x)+C$ **(*C is the constant of integration*)**	O51a
Indefinite Integral of x^a	$\int x^{\alpha}dx=\frac{1}{\alpha+1}x^{\alpha+1}+C \qquad (\alpha\neq -1)$ $\int x^{-1}dx=\int \frac{1}{x}dx=\ln\lvert x\rvert+C$	O51a
Properties of Indefinite Integrals	$\int kf(x)dx=k\int f(x)dx$ (k is a constant) $\int [f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx$ $\int [f(x)-g(x)]dx=\int f(x)dx-\int g(x)dx$	O52a
Indefinite Integral of (ax + b)	When $F'(x)=f(x),\ a\neq 0$, $\int f(ax+b)dx=\frac{1}{a}F(ax+b)+C$	O53a

LEVEL O

<table>
<tr><th>Topic</th><th>Formulas & Notes</th><th>Reference</th></tr>
<tr><td>Indefinite Integrals 2
Indefinite Integrals of Exponential Functions</td><td>$\int e^x dx = e^x + C, \quad \int a^x dx = \frac{a^x}{\ln a} + C$</td><td>O61a</td></tr>
<tr><td>Indefinite Integrals of Trigonometric Functions</td><td>$\int \sin x\, dx = -\cos x + C, \quad \int \cos x\, dx = \sin x + C$
$\int \frac{1}{\cos^2 x} dx = \tan x + C, \quad \int \frac{1}{\sin^2 x} dx = -\frac{1}{\tan x} + C$</td><td>O62a</td></tr>
<tr><td>Integration by Substitution
Integration by Substitution I</td><td>$\int f(x)dx = \int f(g(t))g'(t)dt \qquad (x = g(t))$</td><td>O71a</td></tr>
<tr><td>Integration by Substitution II</td><td>$\int f(g(x))g'(x)dx = \int f(u)du \qquad (g(x) = u)$</td><td>O74a</td></tr>
<tr><td>Indefinite Integral of $\frac{g'(x)}{g(x)}$</td><td>$\int \frac{g'(x)}{g(x)} dx = \ln|g(x)| + C$</td><td>O78a</td></tr>
<tr><td>Integration by Parts</td><td>$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$</td><td>O81a</td></tr>
<tr><td>Definite Integrals</td><td>If $F(x)$ is an indefinite integral (antiderivative) of $f(x)$, then
$\int_a^b f(x)dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$</td><td>O91a</td></tr>
<tr><td>Property of Definite Integrals I</td><td>$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$</td><td>O97a</td></tr>
<tr><td>Integration by Substitution for Definite Integrals</td><td>When $x = g(t)$, if $a = g(\alpha)$ and $b = g(\beta)$, then
$\int_a^b f(x)dx = \int_\alpha^\beta f(g(t))g'(t)dt$ $\quad \begin{array}{c|ccc} x & a & \longrightarrow & b \\ \hline t & \alpha & \longrightarrow & \beta \end{array}$</td><td>O101a</td></tr>
<tr><td>Property of Definite Integrals II</td><td>$\int_b^a f(x)dx = -\int_a^b f(x)dx$</td><td>O102a</td></tr>
<tr><td>Integration of Even/Odd Functions</td><td>When $f(x)$ is an even function, $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$
When $f(x)$ is an odd function, $\int_{-a}^a f(x)dx = 0$</td><td>O107b</td></tr>
<tr><td>Integration by Parts for Definite Integrals and Functions Represented by Definite Integrals
Integration by Parts for Definite Integrals</td><td>$\int_a^b f(x)g'(x)dx = \Big[f(x)g(x)\Big]_a^b - \int_a^b f'(x)g(x)dx$</td><td>O111a</td></tr>
</table>

LEVEL O

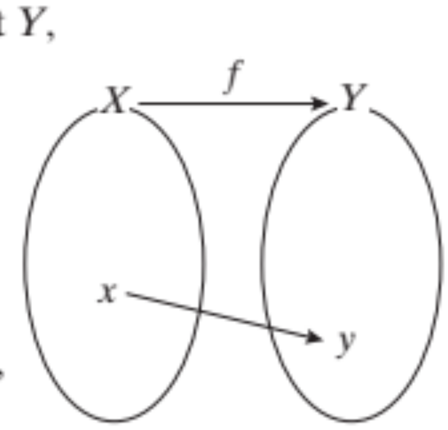
Topic	Formulas & Notes	Reference
Definite Integrals and Differentiation	When a is a constant, $\dfrac{d}{dx}\displaystyle\int_a^x f(t)dt=f(x)$	O115a
Property of Definite Integrals III	$\displaystyle\int_a^a f(x)dx=0$	O116b
Integration by Quadrature and Proof of Inequalities *Definite Integrals and Limits of Sums I*	If the function $f(x)$ is continuous on the interval $[a, b]$, $\displaystyle\lim_{n\to\infty}\sum_{k=0}^{n-1} f(x_k)\Delta x=\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x=\int_a^b f(x)dx$ where $\Delta x=\dfrac{b-a}{n}$, $x_k=a+k\Delta x$	O122a
Definite Integrals and Limits of Sums II	$\displaystyle\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1} f\left(\frac{k}{n}\right)=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)=\int_0^1 f(x)dx$	O122b
Areas *Curve $y=f(x)$ and Area*	On the interval $[a, b]$, when $f(x)\geqslant 0$, $S=\displaystyle\int_a^b f(x)dx$ when $f(x)\leqslant 0$, $S=-\displaystyle\int_a^b f(x)dx$	O131a
Area between Two Curves	On the interval $[a, b]$, when $f(x)\geqslant g(x)$, $S=\displaystyle\int_a^b [f(x)-g(x)]dx$	O132a
Curve $x=g(y)$ and Area	On the interval $c\leqslant y\leqslant d$, when $g(y)\geqslant 0$, $S=\displaystyle\int_c^d g(y)dy$	O134a
Volumes	Let V be the volume of the solid between the planes $x=a$ and $x=b$, where $a<b$. Letting $S(x)$ be the area of the intersection of the plane at x with the solid, $V=\displaystyle\int_a^b S(x)dx$ 	O141b
Volume of Revolution about the x-axis	$V=\pi\displaystyle\int_a^b [f(x)]^2 dx=\pi\int_a^b y^2 dx$	O143a
Volume of Revolution about the y-axis	$V=\pi\displaystyle\int_c^d [g(y)]^2 dy=\pi\int_c^d x^2 dy$	O144a
Length of a Curve Velocity and Distance *Length of a Curve I*	The length L of the curve $x=f(t)$, $y=g(t)$ $(a\leqslant t\leqslant b)$ is $L=\displaystyle\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt=\int_a^b\sqrt{[f'(t)]^2+[g'(t)]^2}\,dt$	O151b

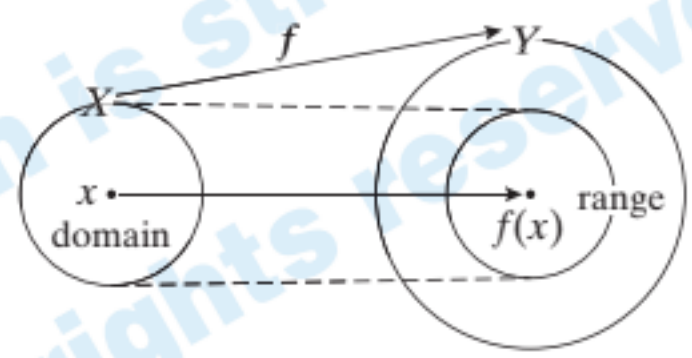
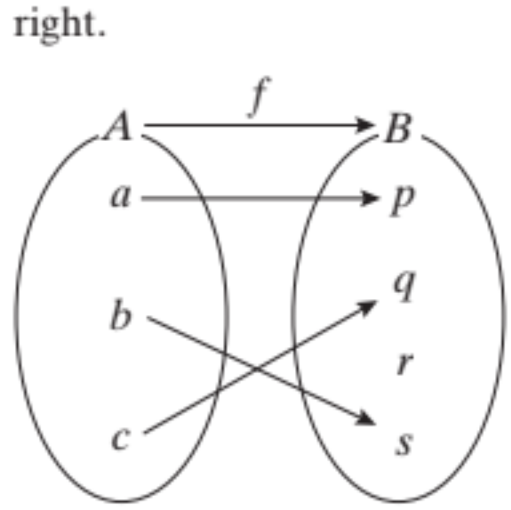
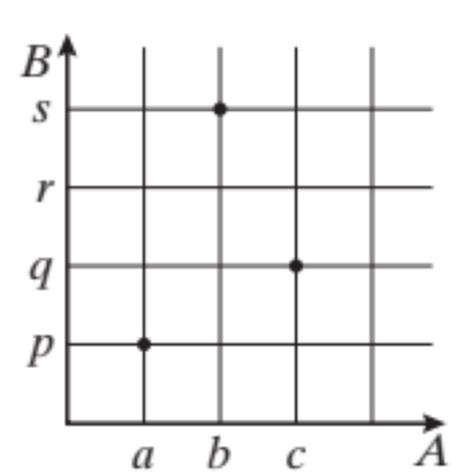
Topic	Formulas & Notes	Reference
Length of a Curve II	The length L of the curve $y=f(x)$ $(a \leqslant x \leqslant b)$ is $$L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int_a^b \sqrt{1+[f'(x)]^2}\,dx$$	O153a
Displacement and Distance of a Point Moving on a Line	$s=\int_a^b v\,dt,\quad l=\int_a^b \lvert v\rvert\,dt$	O155a
Change in the Velocity of a Point Moving on a Line	Let the velocity of point P moving on a number line at time t_0 and t_1 be v_0 and v_1 respectively, and let the acceleration at time t be α. $$v_1=v_0+\int_{t_0}^{t_1} \alpha\,dt$$	O156a
Distance of a Point Moving on a Plane	$l=\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$	O157a
Differential Equations	The differential equation expressed as $f(y)\frac{dy}{dx}=g(x)$ is called a ***separable differential equation***. $\frac{dy}{dx}=\frac{x}{y}$ [Sol] $y\frac{dy}{dx}=x$ ← Rearranging into the form $f(y)\frac{dy}{dx}=g(x)$ Integrating both sides with respect to x, $\int y\,dy=\int x\,dx$ ← LHS $=\int y\frac{dy}{dx}\cdot dx=\int y\,dy$ $\therefore\ \frac{1}{2}y^2=\frac{1}{2}x^2+C_1$ $\therefore\ y^2=x^2+2C_1$ Generally, arbitrary constants do not have coefficients. Let $2C_1=C$. $y^2=x^2+C$ (C is an arbitrary constant)	O164a

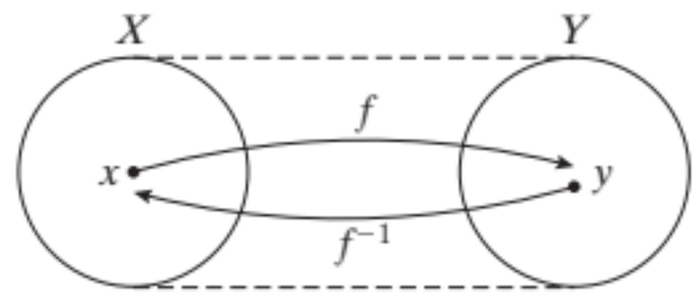
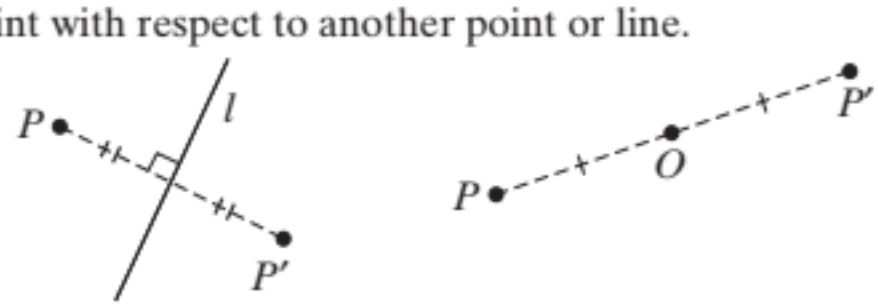
[NOTES]

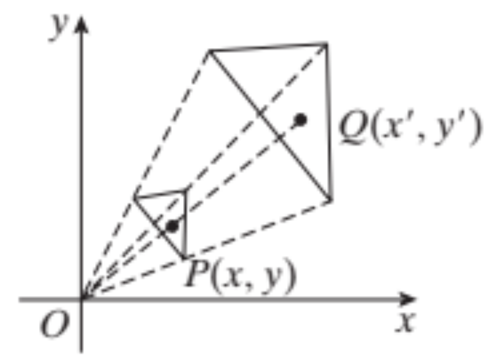
LEVEL X, SECTION XM

Topic	**Formulas & Notes**	**Reference**
Matrix Definitions, Addition, and Subtraction	When referring to a specific matrix, the order of describing the matrix *form* is: (# of rows) × (# of columns) $\begin{array}{cc} & \text{columns} \\ \text{rows} & \begin{bmatrix} 3 & 5 & 8 & 7 \\ 2 & -3 & 7 & 4 \end{bmatrix} \end{array}$	XM1a
	If two matrices A and B are of the same form, and the corresponding components of each matrix are equal, then A and B are equal, and this relationship may be expressed as $A = B$. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \rightleftarrows \begin{matrix} a = p, & b = q \\ c = r, & d = s \end{matrix}$	XM4b
Matrix Sum and Difference	Given two matrices A and B of the same form, the sum $A + B$ and the difference $A - B$ are defined as follows. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$, then: $A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \end{bmatrix}$ $A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} & a_{13} - b_{13} \\ a_{21} - b_{21} & a_{22} - b_{22} & a_{23} - b_{23} \end{bmatrix}$	XM7a
Zero Matrix	A matrix in which all the components are zero is called a *zero matrix* and is designated by the letter O.	XM8a
The Product of a Matrix and a Real Number	Given a matrix A, and a real number k, If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$, then $kA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \end{bmatrix}$	XM9a
Matrix Multiplication	Given that $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, $AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$ Given an $m \times l$ matrix and a $p \times q$ matrix, if $l = p$, then the product of the two matrices is an $m \times q$ matrix.	XM15a
Matrix Multiplication Properties	$k(AB) = (kA)B = A(kB)$ (where k is a real number) $(AB)C = A(BC)$ Associative Property $A(B + C) = AB + AC$ $(A + B)C = AC + BC$ Distributive Property	XM17a
Unit Matrix	$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called a unit matrix. $AE = EA = A$	XM18b

Topic	Formulas & Notes	Reference
Inverse Matrices *Zero Divisor*	When we multiply numbers, if $ab = 0$, then $a = 0$ or $b = 0$. However, when we multiply matrices, if $AB = O$, then $A = O$ or $B = O$ is not always true. Even if $A \neq O$ and $B \neq O$, there may be a case where $AB = O$. In such a case, A and B are both called *zero divisors*.	XM21a
Inverse Matrix	Given that A is a 2×2 matrix, and E is a unit matrix, if there exists a matrix B such that $AB = BA = E$, then B is called the *inverse matrix* of A, and it can be expressed as A^{-1}.	XM22a
Determinant	Given that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$: The quantity $ad - bc$ is called the "*determinant of matrix* A", abbreviated as "det A", where $\det A = ad - bc$. (a) If $\det A \neq 0$, then $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. (b) If $\det A = 0$, then A^{-1} does not exist; A does not have an inverse.	XM24a
Equations and Matrices	Given the simultaneous linear equations: $$\begin{cases} ax + by = p \\ cx + dy = q \end{cases}$$ Letting $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} p \\ q \end{bmatrix}$, and $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $$AX = B$$ When $\det A = ad - bc \neq 0$, $$X = A^{-1}B$$	XM33a
Mapping 1	Given that the elements x of the set X correspond to the elements y of the set Y, The correspondence between x and y is known as the *Mapping from* X *to* Y, and can be denoted by letters, such as f, g, etc. Expressing f as a mapping from X to Y, $f: X \to Y$  A function is a mapping from one set of numbers to another set of numbers.	XM41a

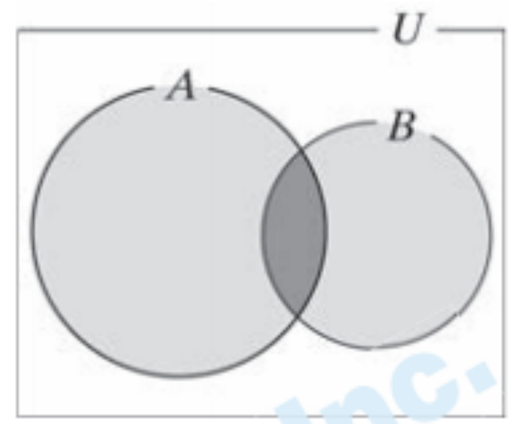
Topic	Formulas & Notes	Reference
	Given $f\colon X \to Y$, When the element x of X corresponds to the element y of Y, y is called the *image* of x through the mapping of f and is expressed as: $$y = f(x)$$	XM41b
Mapping	In general, given two sets, X and Y, in order for mapping from X to Y to occur, each element in X must be mapped to one and only one element in Y.	XM42a
Onto Mapping *Into Mapping*	Given two sets, X and Y, and the mapping $f\colon X \to Y$, if each element of Y is an image of at least one element of X, i.e. when $f(X) = Y$, the mapping is called an *onto mapping*. Other general mappings are called *into mappings*.	XM42b
One-to-one Mapping	Given two sets, X and Y, and the mapping $f\colon X \to Y$, if each element of Y is an image of at most one element of X, the mapping is called a *one-to-one mapping*.	XM43a
	Given the mapping $f\colon X \to Y$, The *domain* of f is set X. The *range* of f is the set of all images of the elements x in X. 	XM44a
	Given the mapping of $f\colon A \to B$, The points (a, p), (b, s), and (c, q) are *ordered pairs*, which can be plotted on a graph. Using a diagram of the *direct product* $A \times B$, The graph of the mapping of $f\colon A \to B$ is shown below on the right. 	XM45a

Topic	Formulas & Notes	Reference
	In general, given the onto mapping $f: X \rightarrow Y$, if the mapping is a one-to-one mapping, then the mapping has an inverse. The inverse is also a mapping, and is expressed by f^{-1}.  **Note:** $(f^{-1})^{-1} = f$	XM46a
Composite Mapping	Given three sets, X, Y, and Z, where f is the mapping from X to Y and g is the mapping from Y to Z, If element x in X corresponds to element y in Y, which corresponds to element z in Z, The correspondence of x to z is the mapping from X to Z. This is called the *Composite Mapping* of f and g, and is expressed by $g \circ f$.	XM48a
	For mapping, generally $f \circ g \neq g \circ f$, but $(f \circ g) \circ h = f \circ (g \circ h)$.	XM48b
Mapping 2 *Transformation*	The set of all points (x, y) on the coordinate plane is expressed as $R \times R$, or R^2. A *transformation* on the coordinate plane is the mapping $f: R^2 \rightarrow R^2$.	XM55a
Parallel Translation	In R^2, when the point (p, q) moves to the point $(p + a, q + b)$, the transformation can be expressed as $f: (x, y) \rightarrow (x + a, y + b)$. This is called a *parallel translation*.	XM55b
Symmetric Translation	A *symmetric translation* is a mapping obtained by translating a point with respect to another point or line. 	XM57a
Transformations 1	Given that $f: (x, y) \rightarrow (x', y')$, the mapping f can be expressed by: $\begin{cases} x' = ax + by \rightarrow \\ y' = cx + dy \rightarrow \end{cases} \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ The mapping f is called a *Linear Transformation*, where $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is known as the matrix of the linear transformation f.	XM61a

Topic	Formulas & Notes	Reference
Similar Transformation	A *Similar Transformation* is the mapping by which $P(x, y)$ is mapped to point $Q(x', y') = (kx, ky)$, where the center is the origin. k is known as the ratio of similitude. 	XM62b
	In a linear transformation, f: $(0, 0) \to (0, 0)$. If f: $(0, 0) \not\to (0, 0)$, The mapping is not a linear transformation.	XM64a
Transformations 2	Given that A is the matrix of the linear transformation f and the inverse matrix A^{-1} of A exists, then the inverse mapping of f is also a linear transformation and its matrix is A^{-1}.	XM71a
	Given that a region is transformed by the matrix of transformation $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the area of the region is increased by a factor of $\lvert ad - bc \rvert$ (where $ad - bc \neq 0$).	XM79a
Transformations 3	If the matrices of the linear transformations f and g are A and B, respectively, then the matrix of the composite transformation $g \circ f$ is BA. Similarly, the matrix of the composite transformation $f \circ g$ is AB.	XM81b
	Given that $\overrightarrow{v'} = f(\vec{v})$, vector $\overrightarrow{v'}$ is the image of vector $\vec{v}$ by the linear transformation f.	XM84b
Linearity of Linear Transformations	• $f(\overrightarrow{p_1} + \overrightarrow{p_2}) = f(\overrightarrow{p_1}) + f(\overrightarrow{p_2})$ • $f(k\vec{p}) = kf(\vec{p})$ • $f(k\overrightarrow{p_1} + l\overrightarrow{p_2}) = kf(\overrightarrow{p_1}) + lf(\overrightarrow{p_2})$	XM85a

[NOTES]

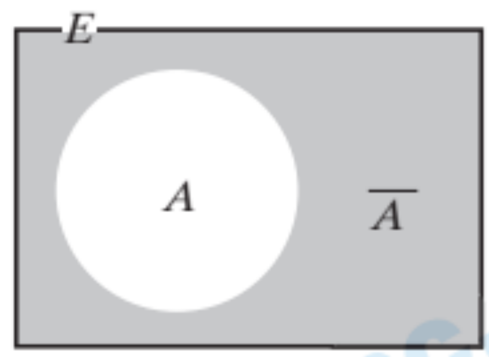
LEVEL X, SECTION XP

Topic	Formulas & Notes	Reference
Introduction to Permutations	A *set* is a collection of objects that satisfy a certain condition, and *elements* are the individual objects that compose a set. Given two sets A and B that share some common elements, • $n(A)$ represents the number of elements in A, and • $n(B)$ represents the number of elements in B. • The *intersection* of A and B, denoted as $A \cap B$, is the set of common elements. (darker shading) • The *union* of A and B, denoted as $A \cup B$, is the set of elements that are contained in either A or B. (lighter and darker shading)  	XP1b
	Given two sets, A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$	XP2a
	Given the two sets, $A = \{a_1, a_2, a_3, \ldots\}$ and $B = \{b_1, b_2, b_3, \ldots\}$, the *product* of A and B is a new set denoted as $A \times B$, and its elements are: $(a_1, b_1), (a_1, b_2), \ldots, (a_2, b_1), (a_2, b_2), \ldots$	XP3a
	Given two sets, A and B, $n(A \times B) = n(A) \times n(B)$	XP3b
Permutations	The number of permutations possible out of n objects, where r objects are used at a time is denoted by ${}_nP_r$. ${}_nP_r = n(n-1)(n-2)(n-3)\ldots(n-r+1)$ $(n \geq r, \quad n, r > 0)$ ${}_nP_n = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$	XP11a
	$n! = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$ (where n is a natural number)	XP11b
	${}_nP_r = \dfrac{n!}{(n-r)!}$ $(n \geq r)$ ${}_nP_n = n!$ $0!$ is defined as 1	XP12a
Permutations and Combinations	The number of permutations possible out of n objects, (where p of one kind are alike, q of another kind are alike, . . . and r of yet another kind are alike) can be expressed as: $\dfrac{n!}{p!\,q!\ldots r!}$ (where $p + q + \cdots + r = n$)	XP21b

LEVEL X, SECTION XP

Topic	Formulas & Notes	Reference
Repeated Permutations	The number of repeated permutations in a set of n objects where r objects are used at a time is: $${}_n\Pi_r = n^r$$	XP23a
Circular Permutations	The ways of arranging different objects in a ring shape are called *circular permutations.* The number of such permutations of n objects is: $$\frac{n!}{n} = (n-1)!$$	XP24a
	The number of permutations of n objects on a *reversible ring* is: $$\frac{(n-1)!}{2}$$	XP25a
Combination	The number of combinations possible out of n objects, where r objects are used at a time is denoted by ${}_nC_r$. $${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$$ $${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!\,(n-r)!}\ (\text{where } {}_nC_0 = 1,\ {}_nC_r = {}_nC_{n-r},\ n \geq r)$$	XP26b
Combinations *Repeated Combination*	A *repeated combination* where n objects are taken r at a time is denoted by ${}_nH_r$. $${}_nH_r = {}_{n+r-1}C_r = \frac{(n+r-1)(n+r-2)\cdots(n+2)(n+1)n}{r!}$$	XP32a
Binomial Theorem	$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$ $$= a^n + {}_nC_1 a^{n-1}b + {}_nC_2 a^{n-2}b^2 + \cdots$$ $$\cdots + {}_nC_r a^{n-r}b^r + \cdots + {}_nC_{n-1}ab^{n-1} + b^n$$	XP42b
	Given that the expression $(a+b+c)^n$ is expanded, the general term is: $$\frac{n!}{p!\,q!\,r!}a^p b^q c^r \qquad (\text{where } p+q+r=n)$$	XP45b
Probability	Letting $p(A)$ be the probability of event A, $n(A)$ be the number of elementary events in A, and $n(E)$ be the number of elementary events in the whole event, $$p(A) = \frac{n(A)}{n(E)}$$	XP52a

LEVEL X, SECTION XP

Topic	Formulas & Notes	Reference
	• $0 \leq p(A) \leq 1$ • The probability of the whole event, E, is 1. • The probability of the empty set, ϕ, is 0.	XP55a
	For any event A, the event that A does not occur is called the *complement event of* A. It is denoted as $\overline{A}$ and is read as "A bar." Thus, $A \cup \overline{A} = E, A \cap \overline{A} = \phi$ and $p(A) + p(\overline{A}) = 1$ (where E is the whole event, and ϕ is the empty event) 	XP55b
	Given two events A and B, $p(A \cup B) = p(A) + p(B) - p(A \cap B)$	XP56a
Addition Theorem for Probabilities	If the two events A and B are mutually exclusive, then $p(A \cup B) = p(A) + p(B)$	XP59a
Conditional Probability *Multiplication Theorem of Conditional Probability*	The probability of events A and B both occuring is: $p(A \cap B) = p(A) \cdot p(B \mid A)$ For three events, A, B, and C, $p(A \cap B \cap C) = p(A) \cdot p(B \cap C \mid A)$ $= p(A) \cdot p(B \mid A) \cdot p(C \mid A \cap B)$ where $(B \cap C \mid A)$ is the conditional probability of $B \cap C$, given A, and $(C \mid A \cap B)$ is the conditional probability of C, given $A \cap B$.	XP62a XP63a
	If events A and B are *independent*, then: $p(B \mid A) = p(B)$ and $p(A \cap B) = p(A) \cdot p(B)$ If events A and B are *dependent*, then: $p(B \mid A) \neq p(B)$ and $p(A \cap B) \neq p(A) \cdot p(B)$	XP64a
	Given events A & B, B & C, and C & A, if each pair consists of two independent events, and if any one of A, B, or C is independent to the other two, then A, B, and C are all independent. Therefore, $p(A \cap B \cap C) = p(A) \cdot p(B) \cdot p(C)$	XP65a
Independent Trials *Theorem of Independent Trials*	Let p be the probability of an event occurring. Given that a trial is repeated n times, the probability of the event occurring exactly r times is: ${}_nC_r \cdot p^r \cdot q^{n-r}$ (where $q = 1 - p$)	XP72a

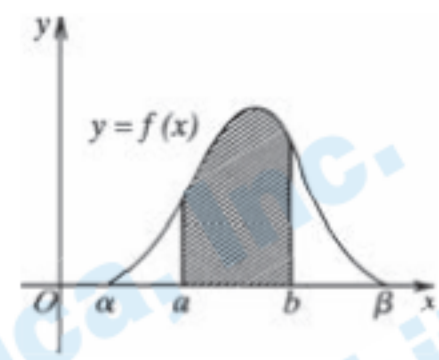
Topic	Formulas & Notes	Reference
Expected value	Given that the variable X takes on one value among $x_1, x_2, \ldots, x_n$, whose respective probabilities are $p_1, p_2, \ldots, p_n$, $$E(X) = \sum_{r=1}^{n} x_r p_r = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$ where $E(X)$ is the *expected value*, or average value, of X.	XP82a

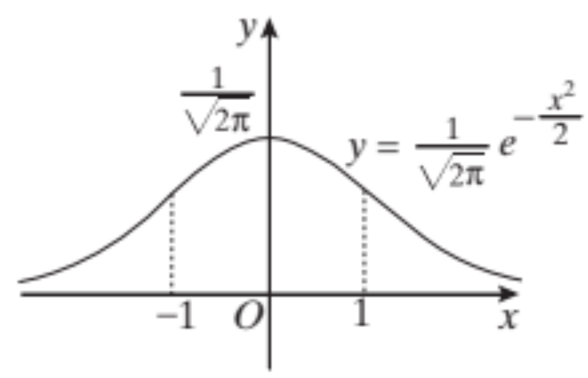
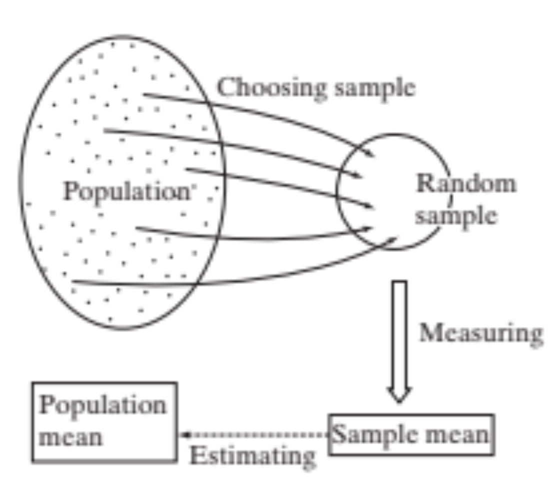
[NOTES]

LEVEL X, SECTION XS

Topic	Formulas & Notes	Reference
Statistics 1	The *probability distribution* of variable X is the arrangement of all of the values of X, and their corresponding probabilities. X is called a *random variable*. The probability that X equals 2 is written as $P(X = 2)$.	XS1a
Mean Value	Letting the probability distribution of the random variable X be: $\begin{array}{|c|c|c|c|c|c|c|} \hline X & x_1 & x_2 & \cdots & x_i & \cdots & x_n \\ \hline P(X) & p_1 & p_2 & \cdots & p_i & \cdots & p_n \\ \hline \end{array}$ Here, the expected value, $E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n = \sum_{i=1}^{n} x_ip_i$ This is also known as the *mean value* of X, denoted as μ.	XS3a
Variance	If X is a random variable and μ is the mean value of X, then the *variance* of X is: $\sigma^2 = E[(X - \mu)^2] = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot p_i$	XS6a
Standard Deviation	$\sigma = \sqrt{\sigma^2}$	XS6a
Statistics 2 *Binomial Distribution*	From the *Theorem of Independent Trials,* If p is the probability of event E occurring in 1 trial, then the probability, P_r, of event E occurring r times in n trials is: $P_r = {}_nC_r \cdot p^r \cdot q^{n-r}$ (where $q = 1 - p$) Letting X be the number of times event E occurs in n trials, The values of X are: $0, 1, 2, \ldots, r, \ldots, n$ Their respective probabilities are: $q^n, {}_nC_1 \cdot p \cdot q^{n-1}, {}_nC_2 \cdot p^2 \cdot q^{n-2}, \ldots, {}_nC_r \cdot p^r \cdot q^{n-r}, \ldots, p^n$ Since these terms are the same as those appearing in the expansion of the binomial theorem, which are: $(q + p)^n = q^n + {}_nC_1 \cdot p \cdot q^{n-1} + \cdots + {}_nC_r \cdot p^r \cdot q^{n-r} + \cdots + p^n,$ This kind of probability distribution is called a *Binomial Distribution*, and is denoted by $B(n, p)$.	XS11a

LEVEL X, SECTION XS

Topic	Formulas & Notes	Reference
	In the binomial distribution $B(n,p)$, the mean value is: $\mu = n \cdot p$, the variance is: $\sigma^2 = n \cdot p \cdot q$, (where $q = 1 - p$) and the standard deviation is: $\sigma = \sqrt{\sigma^2} = \sqrt{n \cdot p \cdot q}$	XS13a
	Given that $Y = aX + b$, where X and Y are random variables, $\mu_Y = a \cdot \mu_X + b, \qquad \sigma_Y^2 = a^2 \cdot \sigma_X^2$	XS15a
Statistics 3	If X is a variable that has continuous real values on the frequency distribution curve $y = f(x)$, the probability that X will be a value in the interval $a \leq x \leq b$ is: $P(a \leq X \leq b) = \int_a^b f(x)dx$ (where $\int_\alpha^\beta f(x)dx = 1$)  In this case X is called a ***continuous random variable***, and $f(x)$ is called **the *probability density function*** of X.	XS21a
	Given that over the interval $[\alpha, \beta]$, the probability density function of X is $f(x)$, then: $\mu = \int_\alpha^\beta x \cdot f(x)\,dx$ $\sigma^2 = \int_\alpha^\beta [x - \mu]^2 \cdot f(x)\,dx$ $\sigma = \sqrt{\sigma^2}$ (where μ is the mean value of X, σ^2 is the variance of X, and σ is the standard deviation of X)	XS23a
Statistics 4 *Normal Distribution*	As n becomes large, the graph of the binomial distribution $B(n, p)$ becomes a symmetric bell curve with the mean value at the center. This curve is a *Normal Distribution Curve*, whose function is: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $\left(\text{where } \int_{-\infty}^{\infty} f(x)dx = 1, \quad \int_{-\infty}^{\infty} xf(x)dx = \mu, \quad \text{and} \quad \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\,dx = \sigma^2\right)$ A distribution of the form given above for a probability density function $f(x)$ is called a *Normal Distribution* and is expressed as $N(\mu, \sigma^2)$, where μ is the mean value and σ is the standard deviation.	XS31b

Topic	**Formulas & Notes**	**Reference**
Standard Normal Distribution	The *Standard Normal Distribution* is the normal distribution with mean value $\mu = 0$ and standard deviation $\sigma = 1$. It is denoted as $N(0, 1)$. The graph of the standard normal distribution is: 	XS32a
	Given that the random variable X follows the standard normal distribution $N(0, 1)$, • $P(X \geq 0) = P(X \leq 0) = 0.5$ • $P(X \geq a) = P(X \leq -a)$ • $P(X = a) = 0$ $\quad\quad \therefore P(X \geq a) = P(X > a)$ • $P(X \geq a) = 1 - P(X \leq a)$ • $P(\lvert X \rvert \leq a) = 2 \cdot P(0 \leq X \leq a)$ $\quad$ (where $a > 0$) • $P(X \geq a) = 0.5 - P(0 \leq X \leq a)$ $\quad$ (where $a > 0$)	XS32a
	Given that the random variable X follows the normal distribution $N(\mu, \sigma^2)$, then the probability distribution of the random variable U is the standard normal distribution, where U is defined as: $$U = \frac{X - \mu}{\sigma}$$	XS36a
Statistics 5	Given that the random variable X follows the binomial distribution $B(n, p)$, X follows the normal distribution $N(np, npq)$ as n becomes large. Therefore, the probability distribution of the random variable U follows the standard normal distribution, where U is defined as: $$U = \frac{X - \mu}{\sigma}$$ (where $\mu = np, \sigma = \sqrt{npq}$, and $q = 1 - p$)	XS41a
Statistics 6	A *Sample Survey* is the statistical method of making estimates about the whole by examining only a randomly chosen part of it. The whole is called the *population* and the randomly chosen part is called the *random sample.* 	XS51a

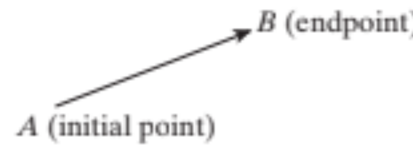
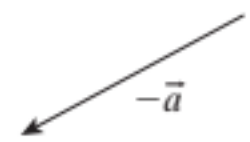
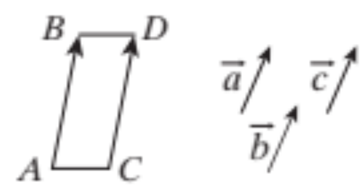
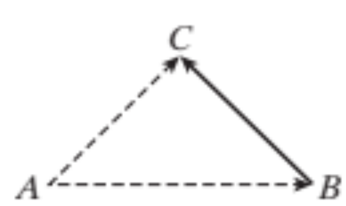
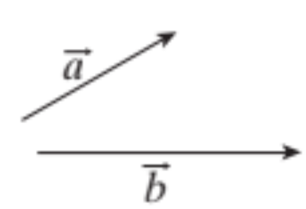
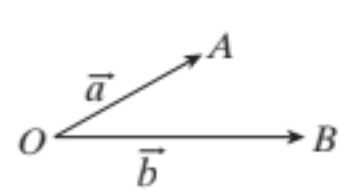
Topic	Formulas & Notes	Reference
	Given that a random sample of n objects is taken from a population whose distribution has a mean value of μ_X with standard deviation σ_X, The sample average, $\overline{X}$, forms a distribution whose mean value is μ_X with standard deviation $\dfrac{\sigma_X}{\sqrt{n}}$. As n becomes larger, the distribution of $\overline{X}$ approaches the normal distribution $N\left(\mu_X, \dfrac{\sigma_X^2}{n}\right)$.	XS52b
	Given a population whose standard deviation is σ and a random sample of n objects whose mean value is $\overline{X}$, The population mean value, μ, can be estimated with 95% and 99% confidence: 95% Confidence Interval: $\overline{X} - \dfrac{1.96\sigma}{\sqrt{n}} < \mu < \overline{X} + \dfrac{1.96\sigma}{\sqrt{n}}$ 99% Confidence Interval: $\overline{X} - \dfrac{2.58\sigma}{\sqrt{n}} < \mu < \overline{X} + \dfrac{2.58\sigma}{\sqrt{n}}$	XS54a
	Given a random sample of n objects, where $\overline{p}$ of the objects have a certain characteristic, The portion p of the population of size N that shares the same characteristic can be estimated with 95% and 99% confidence: 95% Confidence Interval: $\overline{p} - 1.96 \cdot \sqrt{\dfrac{\overline{p}(1-\overline{p})}{n}} < p < \overline{p} + 1.96 \cdot \sqrt{\dfrac{\overline{p}(1-\overline{p})}{n}}$ 99% Confidence Interval: $\overline{p} - 2.58 \cdot \sqrt{\dfrac{\overline{p}(1-\overline{p})}{n}} < p < \overline{p} + 2.58 \cdot \sqrt{\dfrac{\overline{p}(1-\overline{p})}{n}}$ (N must be large in comparison to n, and n also must be sufficiently large.)	XS58a

[NOTES]

Topic	Formulas & Notes	Reference
Statistics 7 *Hypothesis* *Testing*	Given a random sample of n objects, whose mean is $\overline{X}$ and standard deviation is s, Letting σ be the standard deviation of the population, Test the hypothesis that the mean of the population is μ, by letting $u = \dfrac{\overline{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$. (If σ is not given, let $\sigma = s$.) • Using a significance of 5%, If $\lvert u \rvert > 1.96$, then the hypothesis should be rejected. If $\lvert u \rvert \leq 1.96$, then the hypothesis should not be rejected. • Using a significance of 1%, If $\lvert u \rvert > 2.58$, then the hypothesis should be rejected. If $\lvert u \rvert \leq 2.58$, then the hypothesis should not be rejected.	XS64a

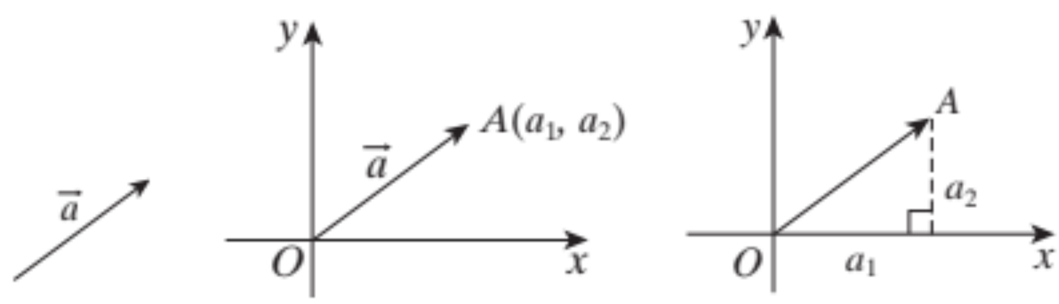
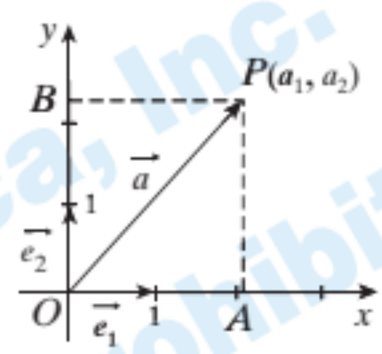
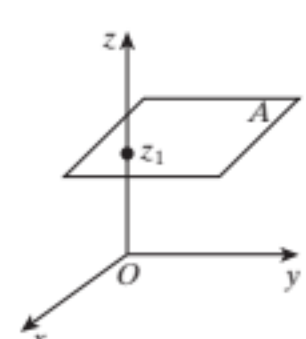
[NOTES]

LEVEL X, SECTION XV

Topic	Formulas & Notes	Reference
Surface Vectors 1	A vector is denoted by a line segment having an *initial point* and an *endpoint*.  The vector on the right is written as: $\overrightarrow{AB}$ Its magnitude is expressed as: $\lvert\overrightarrow{AB}\rvert$ 	XV1a
	A vector of the same magnitude as $\vec{a}$ but opposite direction is written as: $-\vec{a}$  Two vectors are equal if they have the same magnitude and direction. In the figures on the right, $\overrightarrow{AB} = \overrightarrow{CD}$, and $\vec{a} = \vec{b} = \vec{c}$. 	XV1b
	Given two vectors $\vec{a}$ and $\vec{b}$, In order to determine the value of $\vec{a} + \vec{b}$,  Setting $\vec{a}$ to have an initial point O and an endpoint A, and Setting $\vec{b}$ to have initial point A and an endpoint B,  Note that $\vec{b}$ is set to have its initial point at the endpoint of $\vec{a}$. Vector $\overrightarrow{OB}$ represents $\vec{a} + \vec{b}$, where $\overrightarrow{OB} = \vec{a} + \vec{b}$.  Note that $\overrightarrow{OB}$ has the same initial point as vector $\vec{a}$ and the same endpoint as vector $\vec{b}$.	XV2a
	From the figure on the right, we can express the sum of the vectors as: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ Rearranging the equation, $\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$ 	XV4a
	Given two vectors $\vec{a}$ and $\vec{b}$, In order to determine the difference of the two vectors,  Setting $\vec{a}$ to have an initial point O and an endpoint A, and Setting $\vec{b}$ to have initial point O and an endpoint B,  Vector $\overrightarrow{BA}$ represents $\vec{a} - \vec{b}$, where $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \vec{a} - \vec{b}$. Vector $\overrightarrow{AB}$ represents $\vec{b} - \vec{a}$, where $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$.	XV4b
	$\vec{a} - \vec{b}$ may be expressed as $\vec{a} + (-\vec{b})$.	XV5a
Zero Vector	A vector with magnitude 0 is called a *zero vector* and can be expressed as $\vec{0}$. Its direction is undefined.	XV5b

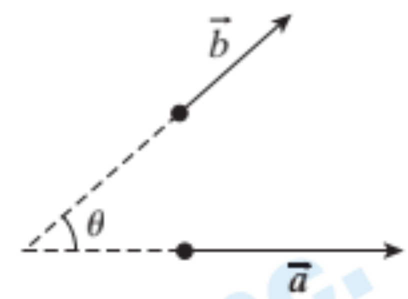
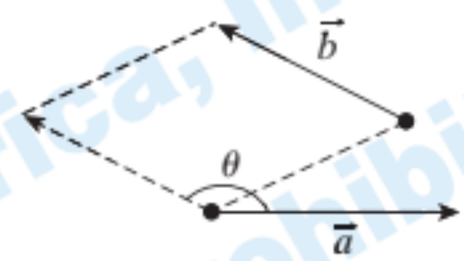
Topic	Formulas & Notes	Reference
Real Number Multiples of Vectors	When a non-zero vector $\vec{a}$ is multiplied by a real number k, the resulting vector $k\vec{a}$ has the following characteristics: (a) When $k > 0$, $k\vec{a}$ points in the same direction as $\vec{a}$, and its magnitude is the product of $\lvert k \rvert$ and the length of $\vec{a}$. (b) When $k < 0$, $k\vec{a}$ points in the opposite direction as $\vec{a}$, and its magnitude is the product of $\lvert k \rvert$ and the length of $\vec{a}$. (c) When $k = 0$, $0\vec{a} = \vec{0}$. Therefore, $k\vec{a}$ is a zero vector. Its magnitude is 0, and its direction is undefined.	XV6b
Properties of Vector Calculations	$(mn)\vec{a} = m(n\vec{a})$ $(m + n)\vec{a} = m\vec{a} + n\vec{a}$ $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$	XV7a
Parallel and Co-Linear Conditions for Vectors	(a) Parallel conditions: Given two nonzero vectors, $\vec{a}$ and $\vec{b}$, $\vec{a} // \vec{b} \rightleftarrows \vec{b} = k\vec{a}$ (where k is a real number) (b) Co-linear conditions: Point P lies on line $AB \rightleftarrows \overrightarrow{AP} = k\overrightarrow{AB}$ (where k is a real number) 	XV8a
Position Vector	Letting O be a fixed point in a plane, the vector from O to a point P can be expressed as $\overrightarrow{OP}$. $\overrightarrow{OP}$ represents the position of point P. Therefore, $\overrightarrow{OP}$ is called the *position vector* of point P. 	XV9a
Surface Vectors 2	A point A whose position vector is $\overrightarrow{OA} = \vec{a}$, can be expressed as $A(\vec{a})$.	XV11b
	Given that a point C divides line segment AB in the ratio of $m : n$, (a) When $m > 0$ and $n > 0$, C internally divides AB in the ratio of $m : n$. (b) When $mn < 0$, C externally divides AB in the ratio of $\lvert m \rvert : \lvert n \rvert$. The position vector of point C which divides line segment AB in the ratio of $m : n$ (as described in the two cases above), is $\vec{c}\,(=\overrightarrow{OC})$ and is expressed as: $\vec{c} = \dfrac{n\vec{a} + m\vec{b}}{m + n}$	XV12a

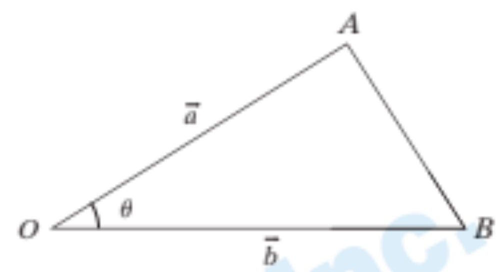
LEVEL X, SECTION XV

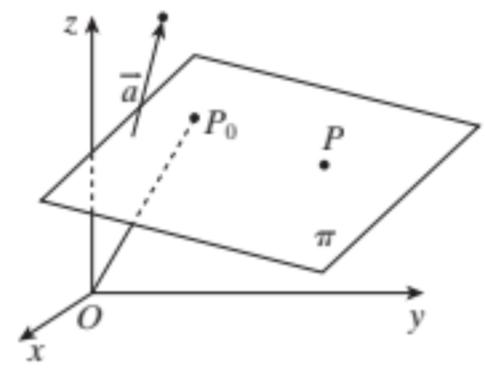
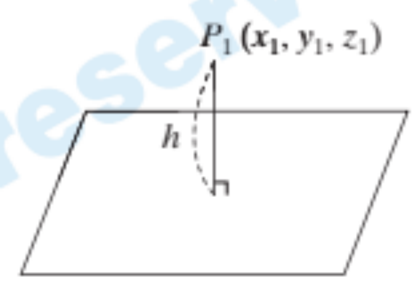
Topic	Formulas & Notes	Reference
Surface Vectors 3	Given that a vector $\vec{a}$ is placed in the coordinate grid as vector $\overrightarrow{OA}$, with components a_1 and a_2, we can express the vector as $\vec{a} = (a_1, a_2)$. When $\vec{a} = (a_1, a_2)$, $\lvert\vec{a}\rvert = \sqrt{{a_1}^2 + {a_2}^2}$ When $\lvert\vec{a}\rvert = 1$, $\vec{a}$ is called a *unit vector*. 	XV25a
Fundamental Vectors	The unit vectors on the x and y axes are $\vec{e_1} = (1, 0)$ and $\vec{e_2} = (0, 1)$, respectively. They are called the *fundamental vectors.* When $\vec{a} = (a_1, a_2)$, $\vec{a} = \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$ $= a_1\vec{e_1} + a_2\vec{e_2}$ This is called the *fundamental vector expression.* 	XV26a
Operations using Components	• $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ • $(a_1, a_2) - (b_1, b_2) = (a_1 - b_1, a_2 - b_2)$ • $m(a_1, a_2) = (ma_1, ma_2)$	XV28a
Coordinates in Space *Parallel Translations*	Given the *parallel translation* of the origin $(0, 0, 0)$ to the point (α, β, γ), an arbitrary point with coordinates (x, y, z) will be moved to: $(x + \alpha, y + \beta, z + \gamma)$	XV42b
	As shown in the figure on the right, plane A is parallel to the xy-plane, and intersects the z-axis. Letting the point of intersection of plane A with the z-axis be point $(0, 0, z_1)$, the equation of plane A can be written as: $z = z_1$ In this case, the equation $z = 0$ represents the xy-plane. 	XV43b

LEVEL X, SECTION XV

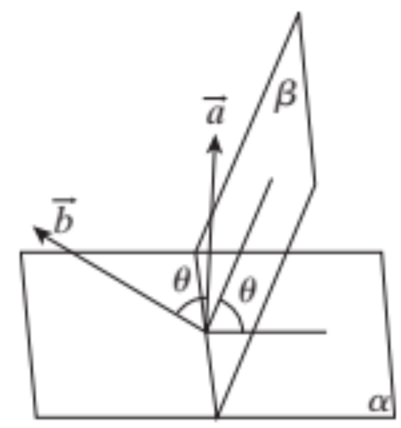
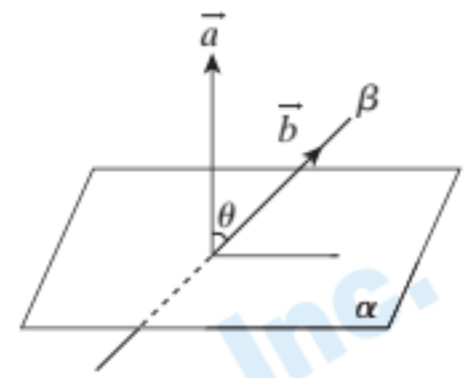
Topic	Formulas & Notes	Reference
Internal and External Dividing Points	Given points $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the points that divide segment AB in the ratio of $m : n$ are as follows: Internally: $\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$ Externally: $\left(\dfrac{mx_2 - nx_1}{m-n}, \dfrac{my_2 - ny_1}{m-n}\right)$ (where $m \neq n$) The midpoint of segment AB can be expressed as: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$	XV44a
	The distance from the origin O to a point $P(x, y, z)$ is expressed as: $OP = \sqrt{x^2 + y^2 + z^2}$	XV46b
Distance between Two Points	The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is expressed as: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	XV46b
Equation of a Sphere	The equation of a sphere with radius r, and center with coordinates (a, b, c), can be expressed as: $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$	XV49a
Vectors in Space	If we reverse the direction of $\overrightarrow{AB}$, $\overrightarrow{BA}$ is called the *inverse vector* of $\overrightarrow{AB}$, and this can be expressed as $\overrightarrow{BA} = -\overrightarrow{AB}$.	XV51a
	$\vec{a} = a\vec{e_1} + b\vec{e_2} + c\vec{e_3}$ (fundamental vector expression) $\vec{a} = (a, b, c)$ (components) The components of $\vec{e_1}, \vec{e_2}, \vec{e_3},$ and $\vec{0}$ are expressed as: $\vec{e_1} = (1, 0, 0)$, $\vec{e_2} = (0, 1, 0)$, $\vec{e_3} = (0, 0, 1)$, $\vec{0} = (0, 0, 0)$	XV52b
Operations with Components	$(a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3)$ $(a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1,\ a_2 - b_2,\ a_3 - b_3)$ $m(a_1, a_2, a_3) = (ma_1, ma_2, ma_3)$ (where m is a real number)	XV53a
	Given that $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, m \neq 0$, $\vec{a} = m\vec{b} \leftrightarrows \vec{a} /\!/ \vec{b}$	XV57a
	Given any two vectors without zero components, $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$, and $\vec{a} /\!/ \vec{b} \rightleftarrows \dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$	XV57a

Topic	Formulas & Notes	Reference
Basics for Inner Products of Vectors *Magnitude of Vectors*	If $\vec{a} = (a_1, a_2, a_3)$, then $\lvert\vec{a}\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$	XV61a
	Given that two vectors $\vec{a}$ and $\vec{b}$ form an angle θ, (where $0° \leq \theta \leq 180°$), letting $\vec{c} = \vec{a} + \vec{b}$, $$\lvert\vec{c}\rvert = \sqrt{\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta}$$	XV65a
Inner Product	Given that two vectors $\vec{a}$ and $\vec{b}$ form an angle θ, where $0° \leq \theta \leq 180°$, $\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$ is called the *inner product* of $\vec{a}$ and $\vec{b}$. This product can be expressed as $\vec{a} \cdot \vec{b}$ or $(\vec{a}, \vec{b})$. $\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$ If $\vec{a} = \vec{0}$ or if $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$   	XV66a
Components and Inner Products of Vectors in Space	If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$, then $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.	XV67b
	The following formula may be used to determine the angle between two surface vectors $\vec{a}$ and $\vec{b}$. $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ $\quad (0° \leq \theta \leq 180°)$	XV69a
Inner Products of Vectors	$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert} = \dfrac{a_1b_1 + a_2b_2}{\sqrt{a_1^2 + a_2^2}\sqrt{b_1^2 + b_2^2}}$ $\quad (0° \leq \theta \leq 180°)$	XV71a
	If two vectors $\vec{a}$ and $\vec{b}$ are perpendicular, then their inner product is zero. If the inner product of two vectors $\vec{a}$ and $\vec{b}$ is zero, then the vectors are perpendicular. $\vec{a} \perp \vec{b} \leftrightarrows \vec{a} \cdot \vec{b} = 0 \quad$ (where $\lvert\vec{a}\rvert\lvert\vec{b}\rvert \neq 0$)	XV72a
	Given two vectors $\vec{a}$ and $\vec{b}$, The sum: $\vec{a} + \vec{b}$ is a vector. The difference: $\vec{a} - \vec{b}$ is a vector. The inner product: $\vec{a} \cdot \vec{b}$ is not a vector. It is a *scalar*.	XV76a

Topic	Formulas & Notes	Reference
Properties of Vectors	$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ $\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ $\vec{a}\cdot(\vec{b}-\vec{c})=\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}$ $(\vec{a}\cdot\vec{b})\vec{c}\neq\vec{a}(\vec{b}\cdot\vec{c})$ $\vec{a}\cdot\vec{b}\cdot\vec{c}$ has no meaning	XV76b
	$\vec{a}\cdot\vec{a}=\lvert\vec{a}\rvert^2$ $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=\lvert\vec{a}+\vec{b}\rvert^2$	XV77a
Applications and Summary of Vectors	Letting $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$ The area of $\triangle OAB$ is: $S=\frac{1}{2}\sqrt{\lvert\vec{a}\rvert^2\lvert\vec{b}\rvert^2-(\vec{a}\cdot\vec{b})^2}$ 	XV83a
Area of a Parallelogram	$S=\sqrt{\lvert\vec{a}\rvert^2\lvert\vec{b}\rvert^2-(\vec{a}\cdot\vec{b})^2}$	XV84a
Vectors and Figures	Given two points $A(\vec{a})$ and $B(\vec{b})$: The position vector $\vec{p}$ of point P, where P internally divides line segment AB in the ratio of $m:n$, is expressed as: $\vec{p}=\dfrac{m\vec{b}+n\vec{a}}{m+n}$ The position vector $\vec{q}$ of point Q, where Q externally divides line segment AB in the ratio of $m:n$, is expressed as: $\vec{q}=\dfrac{m\vec{b}-n\vec{a}}{m-n}$ (where $m\neq n$)	XV93a
	$\vec{a}\cdot\vec{b}=0\rightleftarrows\vec{a}\perp\vec{b}$ (Note: When $\vec{a}$ or $\vec{b}$ are non-zero vectors, if $\vec{a}\cdot\vec{b}=0$, then $\vec{a}\perp\vec{b}$.)	XV97a
Equations of Lines, Planes, and Figures 1	The equation of a line passing through a point $P_0(x_0, y_0, z_0)$ and parallel to a vector $\vec{a}=(a, b, c)$ is expressed as: $\dfrac{x-x_0}{a}=\dfrac{y-y_0}{b}=\dfrac{z-z_0}{c}$ $\vec{a}$ is called a direction vector.	XV103a
	The equation of the line passing through two points (x_0, y_0, z_0) and (x_1, y_1, z_1) can be expressed as: $\dfrac{x-x_0}{x_1-x_0}=\dfrac{y-y_0}{y_1-y_0}=\dfrac{z-z_0}{z_1-z_0}$	XV106b

Topic	Formulas & Notes	Reference
Equations of Lines, Planes, and Figures 2	In the diagram shown below, plane π passes through points P and P_0, and is perpendicular to vector $\vec{a}$. Given that $\vec{p_0}$ and $\vec{p}$ are the position vectors for P_0 and P, respectively,  $\overrightarrow{P_0P} = \vec{p} - \vec{p_0}$ Since $\overrightarrow{P_0P}$ is perpendicular to $\vec{a}$, $\vec{a} \cdot \left(\vec{p} - \vec{p_0}\right) = 0$ This is known as the *vector equation* of the plane.	XV111a
	Given that a plane passes through point $P_0(x_0, y_0, z_0)$ and is perpendicular to vector $\vec{a} = (a, b, c)$, the equation of the plane is expressed as: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$	XV111b
	Given that the distance from the plane to the origin is d, the equation of the plane is: $lx + my + nz = d \qquad (\text{when } l^2 + m^2 + n^2 = 1)$ This is known as the *standard form*.	XV115b
	Given a point $P_1(x_1, y_1, z_1)$ and plane $ax + by + cz + d = 0$, the length of the perpendicular from the point to the plane is: $h = \dfrac{\lvert ax_1 + by_1 + cz_1 + d \rvert}{\sqrt{a^2 + b^2 + c^2}}$ 	XV117a
Equations of Lines, Planes, and Figures 4	The angle formed by the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is the same as the angle formed by the two vectors $\vec{a} = (a_1, b_1)$ and $\vec{b} = (a_2, b_2)$, since $\vec{a}$ and $\vec{b}$ are perpendicular to the two lines respectively.	XV131a
	Letting α be the angle formed by the two vectors, If $\alpha > 90°$, the angle formed by the two lines is $(180° - \alpha)$.	XV131b

[NOTES]

Topic	Formulas & Notes	Reference
	Given two planes α and β, The angle formed by the two planes is the same as the angle formed by the two vectors $\vec{a}$ and $\vec{b}$, where vectors $\vec{a}$ and $\vec{b}$ are the normal vectors of planes α and β, respectively. 	XV132a
	Given plane α and line β, The angle formed by the plane and the line is the complement of the angle formed by the two vectors $\vec{a}$ and $\vec{b}$, where vector $\vec{a}$ is the normal vector of plane α and vector $\vec{b}$ is the direction vector of line β. 	XV133a
	Given a curved surface S and a point $P(\vec{p})$, the *vector equation* of S is the condition for P to lie on S and is expressed as an equality in terms of the position vector, $\vec{p}$. Given the vector equation of S, to determine the equation of the surface, substitute $\vec{p} = (x, y, z)$ into the vector equation.	XV136a

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